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IN

PLANE GEOMETRY

[REVISED]

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BOSTON
THE PALMER COMPANY
1922

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CONTENTS

STRAIGHT LINES

CHAPTER PAGE

I. FIRST PRINCIPLES 1

II. EQUAL ANGLES 4

III. ANGLE-SUMS 6

IV. CONGRUENT TRIANGLES 8

V. EQUALITY FROM CONGRUENT TRIANGLES . 11

VI. PARALLEL LINES 15

VII. EQUALITY FROM PARALLEL LINES . . 18

VIII. PERPENDICULAR LINES 21

IX. INEQUALITY 23

X. LOCI 25

CIRCLES

XI. CIRCLES AND TANGENTS 33

XII. EQUALITY OF ARCS AND CHORDS . . 35

XIII. INEQUALITY OF ARCS AND CHORDS . . 37

XIV. MEASUREMENT OF ANGLES 39

XV. REGULAR POLYGONS 42

PROPORTION

XVI. PROPORTION FROM PARALLEL LINES . . 51

XVII. SIMILAR TRIANGLES 53

XVIII. PROPORTION FROM SIMILAR TRIANGLES . 55

XIX. THE PYTHAGOREAN THEOREM . . . 57

XX. SIMILAR POLYGONS AND CIRCLES . . . 59

AREAS

CHAPTER PAGE
XXI. AREAS OF SIMPLE FIGURES 66
XXII. EQUALITY OF AREAS 68
XXIII. PROPORTION IN AREAS 70
XXIV. MENSURATION OF THE TRIANGLE . . 73
XXV. FURTHER RATIOS 76

MISCELLANEOUS EXERCISES 77
NOTES 86
SUMMARIES OF METHODS 102
DEFINITIONS 112

PREFACE

The special aim of this book is to increase emphasis on analysis, classification, and determination of method in geometry. The pupil is to be taught not the proof but how to find the proof. What is to be done? Then, what are the common methods of doing it?

For this purpose the propositions are arranged not, as usual, according to their conditions, but by their conclusions. The thing to be proved determines the class of proof. The given conditions determine the particular method. Consequently the principal theorems that prove any one thing, such as equality, parallel lines, etc., are grouped in one chapter. The basic principle, upon which all theorems in each chapter depend, is placed conspicuously at the head of that chapter. A Summary of Methods for that group is at the end of that chapter, and this is followed by selected exercises applying these methods. The most important theorems are marked by heavy type and are often followed by a note for emphasis.

This arrangement gives systematic drill in analysis and classification. The pupil soon begins to see that for doing certain things there are certain successful methods. He places a theorem in its class; he recalls the methods for that class; then selects the method for this particular case. He learns one group of methods before taking up another, and he knows the basic principles in each.

Another point of originality is in the explanation of the method of proof just before the proof. The plan is made clear at the beginning, and this fact reduces guessing and memorizing to the minimum.

I gladly give credit to Professor C. H. Currier of Brown University for many valuable suggestions.

IMPORTANT FEATURES OF THIS BOOK

1. Syllabus plan; proofs of theorems to be found in the class.
2. Every definition suggested as needed; these also have to be found, but need not be written.
3. Propositions arranged in groups, each group with a single purpose.
4. Emphasis upon fundamental methods in each group.
5. Summaries of methods in each group.
6. Exercises applying these summaries.
7. Comprehensive sets of review questions on definitions, fundamental principles, and methods.
8. Notes giving a general survey of Geometry, a general survey of each chapter, and methods of proof for each theorem. In some cases special proofs are given.
9. Dictionary of geometrical terms, answering all questions on definitions.
10. It follows closely the recommendations of the National Committee, and is suitable for both elementary and advanced classes.
11. It is especially adapted for reviews and can be used with any text-book. Only seven proofs may not fit the logical order of the class text-book, and these are explained in the Notes, page 86. These notes should be consulted regularly. Pages 83, 84, and 85 are recommended for drill immediately before college examinations.

12. It is ideal for beginning classes if the teacher believes that the pupil should be given "the notion that he is discovering for himself that which he is being taught." Here no other book is needed, but the proof of each theorem marked with an asterisk should be written in a notebook. This work is not harder for the pupil and does not take more time.

SUGGESTED CLASSROOM PROCEDURE

Drill with books open:

1. Stating definitions, summaries, etc., with illustrations.
2. Stating a method of proof for each theorem.
3. Stating a method and outline of proof for each theorem.
4. Stating a method and complete proof for each theorem.
   The work should be distinctly heuristic. Let the students find out everything if possible.

A plan that gives good results is:

1. The theorem is read very carefully. The figure without aid lines is on the blackboard.
2. What is given? First pupil answers.
3. What is to be proved? Second pupil answers.
4. What are methods of proving this thing? Third pupil answers.
5. What does the first method require?
6. Can we get this requirement from the given conditions? Aid lines should be drawn when needed.
7. If not, try another method similarly.
8. After a method has been adopted, the outline or proof is built up, each student taking his turn.

Many theorems not marked with an asterisk may be treated as exercises.

All exercises need not be taken by any one class.

Model theorem, Article 41.

1. Pupils read the theorem to themselves.
2. What is given? $l \parallel l'$ and interior angles u and v on the same side of t.
3. What is to be proved? $\angle u + \angle v =$ 2rt. $\measuredangle$.
4. What are the methods of proving this kind of thing? The basic principle, Art. 26. (State it.)
5. What does this method require? One straight line meeting another straight line, making two adjacent angles.
6. Can we get this requirement from the given conditions? Yes, more than once. Put x for the interior angle adjacent to v.

Proof: First statement, $\angle x + \angle v =$ 2rt. $\measuredangle$. Reason, Art. 26 in words. Second statement, $\angle u = \angle x$. Reason, Art. 37 in words. Third statement, $\angle u + \angle v =$ 2rt. $\measuredangle$. Reason, Article 7b (p. 103).

See what the notes have to say.

In a beginning class each pupil should write down each step of the proof as soon as it is stated orally. This proof should be put in a notebook during a study period. Prepared loose leaf geometry sheets are very convenient for this purpose. See the arrangement of Art. 264, p. 97.

RESULTS CLAIMED BY ITS USE

1. Better view of geometry as a whole; better view of each topic by itself.
2. Increased power of concentration.
3. Increased power of analysis, classification, and method.
4. Increased power of building up a proof.
5. More interest.
6. After a few weeks the pupil can go ahead more rapidly than by the old method.

DRILL BOOK IN PLANE GEOMETRY

STRAIGHT LINES

CHAPTER I

FIRST PRINCIPLES

1. Explain **Solids, Surfaces, Lines, Points, Geometrical Figures.**

2. Explain **Position, Form,** and **Size.** What does Geometry investigate?

3. Explain **Straight** line.

4. Explain **Plain** surface; **Plane Geometry.**

5. Explain **Angle.** What kind of motion does it represent?

6. How can lines or angles be proved equal at first?

7. State six general axioms of equality.

8. Define **Straight Angle.**

9. Define **Adjacent Angles.**

10. Define **Right Angle** and **Perpendicular.**

11. Define **Acute Angle** and **Obtuse Angle.**

12. Define **Supplementary** angles.

13. Define **Complementary** angles.

14. Define **Vertical** angles.

15. Define **Equal** figures. Define **Similar** figures. Define **Congruent** figures.

16. Explain **Superposition.**

17. Define **Parallel Lines.**

18. Define **Triangle.**

19. Define **Axiom.** Define **Theorem.**

20. *Only one straight line can be drawn through two given points.* (Axiom.)

21. *Through a given point, only one straight line can be drawn, making a given angle with a given straight line, and having this angle on a given side of each line.* (Two cases.)

22. *Two straight lines can intersect in only one point.*

23. *All straight angles are equal.*

24. *Through a given point, only one straight line can be drawn perpendicular to a given line.* (Two cases.)

25. *Through a given point only one straight line can be drawn parallel to a given straight line.*

26. *If one straight line meets another straight line, the two adjacent angles are supplementary.*

27. *If two adjacent angles are supplementary, their exterior sides form a straight line.*

SUMMARY

28. Only one straight line can be drawn under certain conditions. State four methods.

State one principle of angle-sums.

State a method of proving two straight lines lie in one straight line.

29. EXERCISES

1. Illustrate by a figure:

- *a.* A straight line extended its own length.
- *b.* A straight line bisected.
- *c.* The sum of two straight lines.

d. The difference of two straight lines.
e. Two adjacent angles.
f. An angle doubled.
g. An angle bisected.
h. The sum of two angles.
i. The difference of two angles.
j. Two perpendicular lines.
k. Two oblique lines.

2. Draw two intersecting straight lines. Name pairs of adjacent angles. Why are they adjacent?

3. Name pairs of supplementary angles. Why are they supplementary?

4. Make the definition of supplementary adjacent angles. Are supplementary angles always adjacent?

5. Name two angles that are supplements of the same angle. What can be said of these two angles?

6. Name pairs of vertical angles. Why are they vertical?

7. How many degrees in an angle that is ½ of a right angle? ⅓ of a right angle? ⅔ of a straight angle? ⅔ of a right angle? ⅚ of two right angles?

8. What is the complement of 15° ? 60° 30′ ? 89° 30′ 15″ ?

9. What is the supplement of 60° ? 100° 50′? 120° 20′ 20″ ?

CHAPTER II

EQUAL ANGLES

The basic method is:
All straight angles are equal. (23.)

---

30. *All right angles are equal.*

31. *Complements of the same or equal angles are equal.*

32. *Supplements of the same or equal angles are equal.*

*33. *If two straight lines intersect, the vertical angles are equal.*

34. Define **Corresponding Angles; Alternate Interior Angles.**

35. *If two straight lines are crossed by a transversal making the corresponding angles equal, the lines are parallel.*

36. *If two parallel lines are crossed by a transversal, the corresponding angles are equal.*

*37. *If two parallel lines are crossed by a transversal, the alternate interior angles are equal.*

*38. *If two straight lines are crossed by a transversal making the alternate interior angles equal, the lines are parallel.*

SUMMARY

39. State seven methods of proving angles equal.
State two methods of proving lines parallel.

40. EXERCISES

1. Draw two parallel lines crossed by a third straight line. Name pairs of equal angles. Why are they equal?

2. In the figure of ex. 1, if one angle equals 60°, what is each of the other three?

3. In the same figure, if one angle is 119° 30′, what is each of the other three?

4. If an angle equals its complement, how many degrees are in the angle?

NOTE. Let $x =$ the angle; then $x =$ complement; then $x + x =$?

5. If an angle equals one-half its complement, how many degrees in the angle?

6. If an angle equals twice its complement, how many degrees in the angle?

7. If an angle equals its supplement, how many degrees in the angle?

8. If an angle equals one-half its supplement, find the angle.

9. If an angle equals twice its supplement, find the angle.

10. Of two complementary angles, one is five times the other. How many degrees in each?

11. Of two supplementary angles, one is five-sevenths of the other. Find each angle.

12. If the sum of three equal angles equals two right angles, how many degrees in each?

13. Divide a line, 84 units in length, into two parts in the ratio 4 to 3.

NOTE. Let $4x$ be one part and $3x$ be the other.

14. Divide a line, 121 units in length, into two parts in the ratio 7 to 4.

15. In the figure of article 33, angle 1 + angle 2 = ? Angle 3 + angle 4 = ? Angle 1 + angle 2 + angle 3 + angle 4 = ?

Therefore, the sum of all the angles about a point in a plane equals what?

16. If the angular space about a point in a plane is divided into six equal parts, how many degrees in each part?

17. What is the test for congruency in all figures? Are straight lines or angles congruent if they are equal? Is this true of all figures? Are all figures equal if they are congruent?

CHAPTER III

ANGLE-SUMS

The basic method is:

If one straight line meets another straight line, the sum of the two adjacent angles equals two right angles. (Art. 26.)

---

*41. *If two parallel lines are cut by a transversal, the sum of the two interior angles on the same side of the transversal equals two right angles.*

*42. **The sum of the angles of a triangle equals two right angles.**

*43. *An exterior angle of a triangle is equal to the sum of the two opposite interior angles.*

44. *If two angles of one triangle equal two angles of another triangle, the third angles are equal.*

45. *The sum of the angles of a quadrilateral equals four right angles.*

46. Define **Polygon; Regular Polygon.**

47. *Into how many triangles can a polygon of n sides be divided by drawing diagonals from any one vertex?*

*48. *The sum of the interior angles of a polygon is equal to two right angles multiplied by a number two less than the number of sides.*

*49. *The sum of the exterior angles of a polygon, formed by producing each side in succession, is equal to four right angles.*

SUMMARY

50. State the basic theorem of angle-sums.

State the commonest theorem.

State the formula for the sum of the interior angles of a polygon; for an angle of a regular polygon.

51. Exercises

1. The sum of the two acute angles of a right triangle equals one right angle.

2. Each angle of an equiangular triangle is one-third of two right angles.

3. Construct an angle of a triangle, having given the other two.

4. Construct an acute angle of a right triangle, having given the other acute angle.

5. Construct an angle of: 45°; 60°; 30°; 22°30′; 82°30′; 7°30′; 3°45′.

6. The bisectors of a pair of vertical angles lie in the same straight line.

7. If an angle of a triangle is 60° and two of the angles are equal, the other angles are what?

8. Find the sum of the interior angles of a polygon of five sides; six sides; eight sides; twenty sides.

9. The sum of the interior angles of a polygon is 16 right angles. How many sides has the polygon?

10. How many degrees in each angle of a regular polygon of twelve sides?

11. Each angle of an equiangular polygon is 140°. How many sides has the polygon?

12. How many sides has a polygon, the sum of whose interior angles equals twice the sum of the exterior angles?

13. Find the sum of the interior angles of a polygon of 52 sides; 77 sides; 1,002 sides; 4 sides; 3 sides; 2 sides.

14. How many sides has a polygon the sum of whose interior angles is 10 right ∠s? 12 right ∠s? 14 right ∠s? 96 right ∠s?

15. How many sides has a polygon if the sum of the interior angles equals the sum of the exterior angles?

16. How many degrees in one angle of a regular polygon of 5 sides? 6 sides? 8 sides? 10 sides?

17. If the base angle of an isosceles triangle is two-fifths of two right angles, the bisector of that angle divides the triangle into two isosceles triangles.

CHAPTER IV

CONGRUENT TRIANGLES

The two basic methods are:

***52. If two triangles have two sides and the included angle of the one equal respectively to two sides and the included angle of the other, the triangles are congruent.**

***53. If two triangles have two angles and the included side of the one equal respectively to two angles and the included side of the other, the triangles are congruent.**

---

54. What are the corresponding parts in Article 53? What can be said of the corresponding parts?

55. *If two triangles have two angles and a side of the one equal respectively to two angles and the corresponding side of the other, the triangles are congruent.*

56. What are the basic theorems of congruent triangles? How are the other theorems proved?

57. What is a **Right Triangle? Hypotenuse? Arm?**

58. *If two right angles have two arms of the one equal respectively to two arms of the other, the triangles are congruent.*

59. *If two right triangles have a side and an acute angle of the one equal respectively to the corresponding side and acute angle of the other, the triangles are congruent.*

*60. *If two sides of a triangle are equal, the opposite angles are equal.*

*61. *If two right triangles have the hypotenuse and an arm of the one equal respectively to the hypotenuse and an arm of the other, the triangles are congruent.*

***62. If two triangles have three sides of the one equal respectively to three sides of the other, the triangles are congruent.**

63. From Articles 58, 59, and 61 a general theorem for all right triangles can be made:

If two right triangles have a side and other part except the right angle of the one equal respectively to a corresponding side and other part except the right angle of the other, the triangles are congruent.

64. What is an **Altitude of a triangle?** A **Median?** An **Angle-bisector?**

65. Define **Circle; Radius; Arc.**

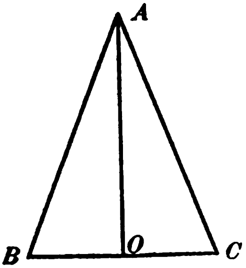
***66.** PROBLEM: *Construct a triangle congruent to a given triangle.*

SUMMARY

67. State four methods of proving triangles congruent, covering all cases.

What is the principal use of congruent triangles?

68. EXERCISES



1. If $AB = AC$ and $\angle BAO = \angle CAO$, prove $\triangle BAO \cong \triangle CAO$.

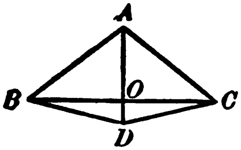
2. If $AB = AC$ and $AO \perp BC$, prove $\triangle BAO \cong \triangle CAO$.

3. If $\angle B = \angle C$ and $AO \perp BC$, prove $\triangle BAO \cong \triangle CAO$.

4. If $AB = AC$ and $BO = CO$, prove $\triangle BAO \cong \triangle CAO$.

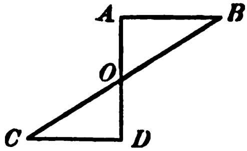
5. If $BO = CO$ and $AO \perp BC$, prove $\triangle BAO \cong \triangle CAO$.

6. If $\angle BAO = \angle CAO$ and $AO \perp BC$, prove $\triangle BAO \cong \triangle CAO$.



7. If $AB = AC$ and $BD = CD$, prove $\triangle ABD \cong \triangle ACD$.

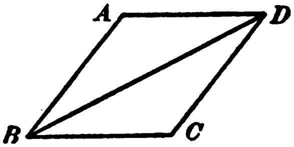
8. If $AB = AC$ and $\angle BAO = \angle CAO$, prove $\triangle ABO \cong \triangle ACO$.



9. If $AD \perp AB$ and CD, and $AO = OD$, prove $\triangle AOB \cong \triangle COD$.

10. If $AO = OD$ and $BO = OC$, prove $\triangle AOB \cong \triangle COD$.

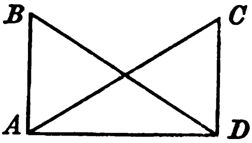
11. If $\angle B = \angle C$ and $BO = OC$, prove $\triangle AOB \cong \triangle COD$.



12. If $AB = CD$ and $AD = BC$, prove $\triangle ABD \cong \triangle DBC$.

13. If $AB = CD$ and $\angle ABD = \angle BDC$, prove $\triangle ABD \cong \triangle DBC$.

14. If $\angle ABD = \angle BDC$ and $\angle ADB = \angle DBC$, prove $\triangle ABD \cong \triangle DBC$.



15. If $AB = CD$, and AB and $CD \perp AD$, prove $\triangle ABD \cong \triangle ACD$.

16. Illustrate by a figure:
- *a.* A triangle with two sides equal.
- *b.* A triangle with three sides equal.
- *c.* The three altitudes of a triangle.
- *d.* The three medians of a triangle.
- *e.* The three angle-bisectors of a triangle.
- *f.* In a triangle with two sides equal, the altitude, the median, and the angle-bisector at the included vertex.
- *g.* In a triangle with three sides equal, the altitude, the median, the angle-bisector at any one vertex.

CHAPTER V

EQUALITY FROM CONGRUENT TRIANGLES

The basic method is:

The corresponding parts of congruent triangles are equal. (Art. 54.)

---

69. Define **Scalene** triangle.

70. Define **Isosceles** triangle.

71. Define **Equilateral** triangle.

72. Define **Equiangular** triangle.

Review Art. 60.

73. *If three sides of a triangle are equal, the three angles are equal.*

*74. *If two angles of a triangle are equal, the opposite sides are equal.*

75. *If three angles of a triangle are equal, the three sides are equal.*

*76. *The perpendicular from the vertex to the base of an isosceles triangle:*

1. *Divides the triangle into two congruent triangles.*
2. *Bisects the angle at the vertex.*

*77. *If equal lines are drawn from a point in a perpendicular to a base line, they cut off equal distances on the base line.*

*78. *If lines are drawn from a point in a perpendicular to a base line cutting off equal distances on the base line, they are equal.*

*79. PROBLEM: *Construct an angle equal to a given angle.*

*80. PROBLEM: *Bisect a given angle.*

81. PROBLEM: *Construct a triangle, having given two sides and the included angle.*

82. Problem: *Construct a triangle, having given two angles and the included side.*

83. What should be memorized? What should not?

84. What is a **Converse** theorem? An **Opposite** theorem?

SUMMARY

85. State two methods of proving lines equal.
State two new methods of proving angles even.
State the commonest method for each.

86. EXERCISES

1. Any point in the bisector of the vertical angle of an isosceles triangle is equidistant from the ends of the base.

2. If the bisector of an angle of a triangle is perpendicular to the opposite side, the triangle is isosceles.

3. The median to the base of an isosceles triangle bisects the vertical angle.

4. If the median to one side of a triangle is perpendicular to that side, the triangle is isosceles.

5. The altitudes upon the sides of an isosceles triangle are equal.

6. If the altitudes upon two sides of a triangle are equal, the triangle is isosceles.

7. In an isosceles triangle, the lines drawn from the ends of the base to the middle of the sides (medians) are equal.

8. If two intersecting lines bisect each other, the opposite lines joining their ends are equal.

9. The bisectors of the base angles of an isosceles triangle are equal.

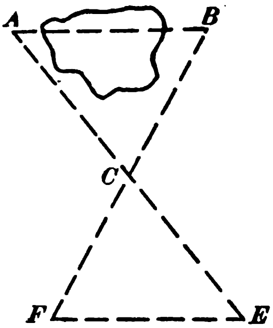
10. If D is in the middle point of side BC of triangle ABC, and BE and CF are lines from B and C perpendicular to AD, produced if necessary, then $BE = CF$.

11. The perpendiculars drawn from the middle point of the base to the sides of an isosceles triangle are equal.

12. State and prove the converse of 11.

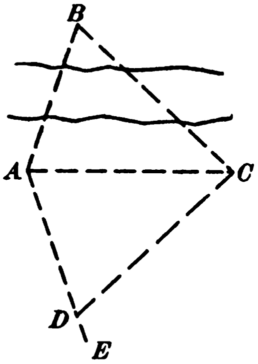
13. A method of finding the distance between two points, *A* and *B*, separated by a pond:

Select a point *C* at one side of the pond, so that *AC* and *BC* can be measured. Extend *AC* its own length to *E*, and *BC* its own length to *F*. *FE* = *AB*. Why?



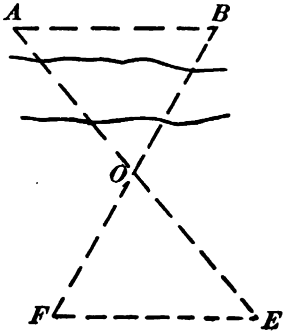
14. A method of finding the distance between two points, *A* and *B*, separated by a stream:

Construct any convenient line *AC*. Measure the angle *BAC* and construct angle *CAE* = *BAC*. Construct ∠*ACD* = ∠ *ACB*. *AD* = *AB*. Why?



15. A method of finding the distance between two points, *A* and *B*, both on the further side of a stream:

Find *OA* and *OB* as in ex. 14; then find *FE* as in ex. 13.



REVIEW QUESTIONS

1. What are the three elements of a geometrical figure?

2. What three qualities of a geometrical figure are discussed and compared?

3. Two figures that have the same size are . . . what?

4. Two figures that have the same form are . . . what?

5. Two figures that have the same form and size are . . . what?

6. How many straight lines can be drawn through two points?

7. What may an angle be said to represent.

8. Upon what does the size of an angle depend?

9. Define Adjacent angles; Straight angles; Right angles; Acute angles; Obtuse angles; Supplementary angles; Complementary angles; Vertical angles.

10. State six general axioms of equality.

11. Define Perpendicular.

12. If one straight line meets another straight line, the sum of the two adjacent angles is . . . what?

13. If one straight line crosses another straight line, the vertical angles are . . . what?

14. If the sum of two adjacent angles equals two right angles, . . . what?

15. What can be said of the supplements of the same angle? Complements of the same angle?

16. Define Triangle; Altitude; Median; Angle-bisector.

17. Define Superposition.

18. What are its three main steps?

19. What is its principal use?

20. What is a Right triangle? Hypotenuse? Arm?

21. How many times was superposition used in proving triangles congruent? In what theorems?

22. How were the other cases of congruent triangles proved?

23. What two theorems of congruent triangles were proved by placing triangles adjacent?

24. Summarize congruent triangles into four cases.

25. If one line is to be proved equal to a second line, what is the usual method?

26. In proving lines equal, which one of the four cases of congruent triangles cannot be used?

27. In proving a line is bisected, what equality is to be proved?

28. In proving an angle is bisected, what equality is to be proved?

29. State seven methods of proving angles equal.

CHAPTER VI

PARALLEL LINES

The two basic methods are:

***35. If two straight lines are cut by a transversal making the corresponding angles equal, the lines are parallel.**

***38. If two straight lines are cut by a transversal making the alternate-interior angles equal, the lines are parallel.**

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87. Define **Parallel Lines.**

88. What are Alternate-interior angles of parallel lines? Corresponding angles?

89. What is the usual method of proof called?

What is the Indirect Method of proof?

State the four steps of the Indirect Method.

***90.** *If two straight lines are cut by a transversal making the two interior angles on the same side of the transversal supplementary, the lines are parallel.*

***91.** *Two straight lines perpendicular to a third straight line are parallel.*

***92.** PROBLEM: *Construct through a point a line parallel to a given line.*

93. State the axiom of parallel lines.

94. *Two straight lines parallel to a third straight line are parallel to each other.*

95. Define **Quadrilateral.** 96. Define **Trapezoid.**

97. Define **Parallelogram.** 98. Define **Rhombus.**

99. Define **Rectangle.** 100. Define **Square.**

*101. *If the opposite sides of a quadrilateral are equal, the figure is a parallelogram.*

*102. *If two opposite sides of a quadrilateral are equal and parallel, the figure is a parallelogram.*

*103. *If the diagonals of a quadrilateral bisect each other, the figure is a parallelogram.*

104. *If the opposite angles of a quadrilateral are equal, the figure is a parallelogram.*

SUMMARY

105. State four methods of proving quadrilaterals parallelograms.

State six methods of proving lines parallel.

106. EXERCISES

1. If two sides of a triangle are produced their own lengths, through their common vertex, the line joining their ends is parallel to the third side.

2. If two parallel lines are cut by a transversal, the bisectors of two alternate-interior angles are parallel.

3. If two parallel lines are cut by a transversal, the bisectors of two corresponding angles are parallel.

4. If two lines are cut by a transversal making the alternate-exterior angles equal, the lines are parallel.

5. If two lines are cut by a transversal making the two exterior angles on the same side of the transversal supplementary, the lines are parallel.

6. If side AC of triangle ABC is bisected at E, and BE is drawn and then extended its own length to F, AF is parallel to BC.

7. If a quadrilateral has all angles right angles, the figure is a parallelogram.

8. In $\triangle ABC$, E and F are the mid-points of AB and AC respectively; EF is extended its own length to H. Prove $AE =$ and $\parallel HC$.

9. In the same figure prove $EBCH$ a parallelogram.

10. In trapezoid $ABCD$, $AD \parallel BC$; E and F are the mid-points of AB and CD respectively; AF is drawn and extended to meet BC extended at H. Prove $AF = FH$.

11. In the same figure, what can be said of EF and BH Why?

12. If two straight lines are cut by a transversal making alternate-interior angles equal, any other transversal makes alternate-interior angles equal.

13. The line bisecting the exterior angle at the vertex of an isosceles triangle is parallel to the base.

14. If on the equal sides of an isosceles triangle equal distances are measured from the vertex, the line joining these points is parallel to the base.

15. If a line is drawn connecting the ends of the bisectors of the base angles of an isosceles triangle, this line is parallel to the base.

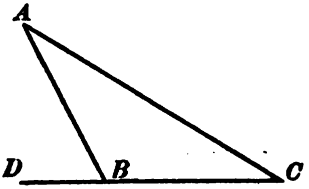
16. If a line is drawn connecting the ends of the perpendiculars from the ends of the base to the sides of an isosceles triangle, this line is parallel to the base.

17. If a line is drawn connecting the ends of the medians to the sides of an isosceles triangle, this line is parallel to the base.

18. If an exterior angle of a triangle equals twice an opposite interior angle, the triangle is isosceles.

19. A method of finding the distance between a moving boat and a distant object such as a lighthouse:

Let A be the lighthouse and C the boat, moving along CD. Find the angle ACD. As the boat moves, this angle increases. When the angle is twice as large, such as ABD, the distance CB, which the boat has gone, equals AB. Why?



CHAPTER VII

EQUALITY FROM PARALLEL LINES

The two basic methods are:

*36. **If two parallel lines are cut by a transversal, the corresponding angles are equal.**

*37. **If two parallel lines are cut by a transversal, the alternate-interior angles are equal.**

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*107. *Angles having their sides parallel each to each are either equal or supplementary.*

*108. *Angles having their sides perpendicular each to each are either equal or supplementary.*

*109. *In any parallelogram, a diagonal makes two congruent triangles; the opposite sides are equal; and the opposite angles are equal.*

110. *The diagonals of a parallelogram bisect each other.*

111. *In any rectangle, the diagonals are equal.*

112. *Two parallelograms are congruent if they have two adjacent sides and the included angle of the one equal respectively to two adjacent sides and the included angle of the other.*

*113. **Parallel lines cutting off equal parts on one transversal cut off equal parts on any other transversal.**

114. PROBLEM: *Divide a straight line into any number of equal parts.*

115. State two methods of proving one thing twice another.

116. *The line joining the middle points of two sides of a triangle is parallel to the third side and equals one-half of it.*

117. *The line joining the middle points of the non-parallel sides of a trapezoid is parallel to the bases and equals one-half their sum.*

SUMMARY

118. State five methods of proving angles equal.
State four additional methods of proving lines equal.

119. EXERCISES

1. Parallel lines included between parallel lines are equal.

2. Two parallel lines are everywhere equally distant.

3. A line drawn through the vertex of an angle, perpendicular to its bisector, makes equal angles with the sides of the angle.

4. The bisector of an angle bisects also its vertical angle.

5. The median to the base of an isosceles triangle bisects the angle at the vertex.

6. If the non-parallel sides of a trapezoid are equal, the base angles are equal.

7. State and prove the converse.

8. If the diagonals of a parallelogram are perpendicular to each other, the figure is equilateral.

9. The diagonals of an equilateral parallelogram bisect the angles.

10. If from any point in the bisector of an angle a parallel to one of the sides be drawn, the bisector, the parallel, and the remaining side form an isosceles triangle.

11. If a line is drawn from the vertex of an isosceles △, parallel to the base, it bisects the exterior angle at the vertex.

12. If the bisector of an exterior angle of a triangle is parallel to the opposite side, the triangle is isosceles.

13. The diagonals of an isosceles trapezoid are equal.

14. If the non-parallel sides of an isosceles trapezoid are extended until they meet, an isosceles triangle is formed.

15. If a diagonal of a parallelogram bisects one angle, the figure is equilateral.

16. The line from the vertex of the right angle of a right triangle to the mid-point of the hypotenuse equals one-half the hypotenuse.

SPECIAL RIGHT TRIANGLES

120. The 60–30 right triangle and the 45° right triangle are useful in numerical exercises.

121. EXERCISES

1. A diagonal of a square makes two 45° right triangles.

2. An altitude of an equilateral triangle makes two 60–30 right triangles.

3. If one angle of a right triangle is 45°, the two arms are equal.

4. If the two arms of a right triangle are equal, each acute angle equals 45°.

5. If angle A of right triangle ABC is 60° and angle C is a right angle, extend AC its own length to D; draw BD. Prove $\angle D = \angle A = \angle ABC$.

6. If one angle of a right triangle is 60°, the hypotenuse is twice the shorter arm.

7. If the hypotenuse of a right triangle is twice the shorter arm, the included angle is 60°.

8. In right triangle ABC, the hypotenuse $AB = 4$ and $\angle A = 60°$; what is AC?

9. In triangle ABC, $AB = 6$ and $\angle A = 30°$; what is the altitude from B to AC?

10. In triangle ABC, $AB = 6$ and $\angle A = 150°$; what is the altitude from B to AC?

11. If each arm of an isosceles triangle is 10 and the altitude is 5, what is the vertex angle?

12. If the base of an isosceles triangle is 8 and the altitude is 4, what is the vertex angle?

13. In triangle ABC, $AC = 7$ and $\angle A = 135°$; the altitude from B meets CA extended at D. If $AD = 5$, what is BD?

14. In triangle ABC, $AB = 4$ and $\angle A = 120°$; the altitude from B meets CA extended at D. What is AD?

EXERCISES ON PARALLELOGRAMS

1. The line joining the mid-points of two opposite sides of a parallelogram cuts off a parallelogram.

2. If E and F are the mid-points of sides AB and CD of parallelogram $ABCD$, then $AECF$ is a parallelogram.

3. The lines joining the mid-points of the sides of a quadrilateral taken in order form a parallelogram.

4. In triangle ABC, D, E, and F are the mid-points respectively of AB, BC, and CA. Prove $ADEF$ a parallelogram.

5. If E and F are the mid-points of AB and CD of quadrilateral $ABCD$; and H and K are the mid-points of diagonals AC and BD, then $EKFH$ is a parallelogram.

6. In triangle ABC, the medians AH and CK meet at O; E is the mid-point of AO, and F of CO. Prove $EKHF$ a parallelogram.

CHAPTER VIII

PERPENDICULAR LINES

The basic method is:

If one straight line meets another straight line, making two adjacent angles equal, the lines are perpendicular. (Art. 10.)

***122. Two points each equally distant from the ends of a line determine the perpendicular bisector of that line.**

***123.** PROBLEM: *Construct the perpendicular bisector of a given straight line.*

***124.** PROBLEM: *Construct a perpendicular to a given straight line from a point in the line.*

***125.** PROBLEM: *Construct a perpendicular to a given straight line from a point outside the line.*

***126.** *A line perpendicular to one of two parallel lines is perpendicular to the other.*

127. *The diagonals of a rhombus bisect each other at right angles.*

SUMMARY

128. State five methods of proving lines perpendicular.

129. EXERCISES

1. The bisector of the vertical angle of an isosceles triangle is perpendicular to the base.

2. The median to the base of an isosceles triangle is perpendicular to the base.

3. The bisectors of two supplementary adjacent angles are perpendicular to each other.

4. If two isosceles triangles have the same base, the line joining their vertices is perpendicular to the base.

5. If a parallelogram has one angle a right angle, the figure is a rectangle.

6. A line parallel to the base of an isosceles triangle is perpendicular to the bisector of the vertex angle.

7. If an angle of a triangle equals the sum of the other two, this angle is a right angle.

8. If a base angle of an isosceles triangle is 45°, the vertex angle is a right angle.

9. If an angle of a triangle is 60° and one including side is double the other, the triangle is a right triangle.

10. The line joining the middle points of the bases of an isosceles trapezoid is perpendicular to the bases.

11. The perpendicular bisector of one base of an isosceles trapezoid is the perpendicular bisector of the other base.

12. If the diagonals of a parallelogram are equal, the figure is a rectangle.

13. If the median to the base of a triangle is equal to one-half the base, the angle at the vertex is a right angle.

14. If the diagonals of a parallelogram are equal and perpendicular, the figure is a square.

15. The lines joining the middle points of the sides of a square, taken in order, form a square.

16. The bisectors of the angles of a parallelogram form a rectangle.

CHAPTER IX

INEQUALITY

The two basic methods are:

130. **The whole is greater than any one of its parts.** (Art. 7*a*.)

131. **A straight line is the shortest line between two points.** (Axiom.)

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132. State six general axioms of inequality.

133. *Supplements or complements of unequal angles are unequal in reverse order.*

*134. *An exterior angle of a triangle is greater than either opposite interior angle.*

*135. *If two sides of a triangle are unequal, the opposite angles are unequal and the greater angle is opposite the greater side.*

*136. *If two angles of a triangle are unequal, the sides opposite are unequal and the greater side is opposite the greater angle.*

137. *The sum of two lines drawn from a point to the ends of a line is greater than the sum of two other lines similarly drawn but enveloped by them.*

138. *If two lines are drawn from a point in a perpendicular, cutting off unequal distances from the foot of the perpendicular, the more remote is the greater.*

139. (Conversely.) *If two unequal lines are drawn from a point in a perpendicular, cutting off distances from the foot of the perpendicular, the greater cuts off the greater distance.*

*140. a. *The perpendicular is the shortest line that can be drawn from a point to a line.*

b. (Conversely.)

***141.** *If two triangles have two sides of the one equal respectively to two sides of the other, but the included angle of the first greater than the included angle of the second, the third side of the first is greater than the third side of the second.*

***142.** *If two triangles have two sides of the one equal respectively to two sides of the other, but the third side of the first greater than the third side of the second, the included angle of the first is greater than the included angle of the second.*

SUMMARY

143. State four methods of proving angles unequal.

144. State seven methods of proving lines unequal.

145. State an additional method of proving lines perpendicular.

146. EXERCISES

1. If point E be taken on side AB of triangle ABC, prove $AB + AC$ greater than $EB + EC$.

2. In equilateral triangle ABC, extend AC to D. Prove angle ABD greater than angle ADB; $\angle ABD > \angle A$; $\angle A > \angle D$.

3. In equilateral triangle ABC, extend AC to D. Prove AD greater than BD, and BD greater than AB.

4. If in triangle ABC the median AE be drawn, making the acute angle AEB, prove AB less than AC.

5. H is a point on the base BC of isosceles triangle ABC. D is a point on the side AB extended through A. F is a point on BC extended through C. K is a point outside of the triangle, to the right of A. Prove AB greater than AH.

6. In the same figure, prove AF greater than AB.

7. In the same figure, prove BD greater than DC.

8. In the same figure, prove BK greater than KC.

9. One side of a triangle is less than the sum of the other two and greater than their difference.

10. The sum of the lines drawn from a point within a triangle to its vertices is less than the perimeter of the triangle; is greater than one-half the perimeter.

CHAPTER X

LOCI

The basic principle is the definition:

The locus of points fulfilling certain conditions is the line or lines that contain all points under these conditions and no others.

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147. Define **Locus.**

148. *a.* Place a point one inch from a given point O. Place a second point one inch from O. Place many points in the same way.

Draw a line through these points. What kind of line is it?

This line is called the locus of points one inch from O.

b. Place a point one inch from a given straight line AB. Place a second point one inch from AB. Place many points in the same way.

Draw lines through these points. What kind of lines are they?

These lines form the locus of points one inch from AB.

c. Place a point midway between two parallel lines AB and CD. Place a second point in the same way. Place many points.

Draw a line through these points. What kind of line is it?

This line is the locus of points equally distant from AB and CD.

149. The general statement of 148*a* is:

The locus of points at a given distance from a given point is a circle with the given point as center and the given distance as radius.

150. The general statement of 148*b* is:

The locus of points at a given distance from a given line consists of two lines, parallel to the given line, on opposite sides of it, and at the given distance from it.

151. The general statement of 148*c* is:

The locus of points equidistant from two parallel lines is a parallel line, midway between the two given lines.

***152. The locus of points equidistant from the ends of a line is the perpendicular bisector of that line.**

***153. The locus of points equidistant from two intersecting lines is the pair of lines bisecting the angles.**

SUMMARY

154. *a.* State a method of finding the locus of points under given conditions.

b. State a method of finding a point under two given conditions.

c. State a method of proving a line is the locus of points under given conditions.

d. State the five fundamental locus figures.

e. State two methods of proving a line passes through a given point.

155. EXERCISES

1. Find a point at a given distance from a given line, and equidistant from two lines that meet.

2. Find a point at a given distance from a given line, and equidistant from two parallel lines. When will there be no case?

3. Find a point equidistant from two lines that meet, and equidistant from two parallel lines.

4. Find a point equidistant from two given points, and equidistant from two given intersecting lines

5. Find a point equidistant from two given points, and equidistant from two parallel lines.

6. In a given line, find a point equidistant from two given points. When is there no case?

7. In a given line, find a point at a given distance from a given line.

8. In a given line, find a point equidistant from two intersecting lines.

9. In a given line, find a point equidistant from two parallel lines.

10. Find a point equidistant from three given points, not in a straight line.

11. Find a point equidistant from three given non-parallel lines.

12. Any two angle-bisectors of a triangle meet in a point equidistant from the three sides.

13. The three angle-bisectors of a triangle meet in a common point.

14. Any two perpendicular bisectors of the sides of a triangle meet in a point equidistant from the three vertices.

15. The three perpendicular bisectors of the sides of a triangle meet in a common point.

16. The three altitudes of a triangle meet in a common point.

17. Any two medians of a triangle meet in a point, which divides each median into two parts, one double the other.

18. The three medians of a triangle meet at a common point.

19. Find the locus of the vertices of triangles, having the same base and a given altitude.

20. Find the locus of the middle points of lines drawn from a given point to a given line. Prove the result.

156. State the four steps in Analysis of Theorems.

State the four cases of concurrent lines in triangles.

REVIEW QUESTIONS

1. Summarize congruent triangles into four cases.

2. What is the principal use of congruent triangles?

3. What is a converse theorem? An opposite theorem?

4. Define Isosceles triangle; Scalene; Equilateral.

5. Give the definition of parallel lines, and the axiom.

6. Make three other statements beginning with "only one straight line."

7. What is the Indirect Method of proof?

8. Define Quadrilateral; Trapezoid; Parallelogram; Rhombus; Rectangle; Square.

9. State four methods of proving a quadrilateral is a parallelogram.

10. State seven methods of proving lines parallel.

11. State two methods of proving one thing twice another.

12. Define Polygon; Regular polygon.

13. State three principles of angle-sums in triangles or polygons.

14. State six methods of proving lines perpendicular.

15. State the basic axiom of all inequality.

16. State the axiom of inequality for straight lines.

17. State four methods of proving angles unequal.

18. State seven methods of proving lines unequal.

19. What are the five fundamental locus figures?

20. Define Locus.

21. State a method of finding the locus of points under given conditions.

22. State a method of finding a point under two given conditions.

23. State a method of proving a line is the locus of points under given conditions.

24. State four steps in an Analysis of Theorems.

25. What class of lines do not meet?

26. What class of lines do meet?

27. In Chapter V, the two main steps in most of the proofs are . . . what?

28. If two straight lines are cut by a transversal making the alternate-interior angles equal, . . . what?

29. State the converse.

30. What are the two commonest methods of proving lines parallel?

31. Parallel lines cutting off equal parts on one transversal, . . . what?

32. What is the fundamental principle of angle-sums?

33. What is the commonest theorem of angle-sums?

34. What is the fundamental principle of perpendicular lines?

35. What is the commonest theorem?

36. The theorems of unequal angles are usually proved from what axiom?

37. The theorems of unequal lines are often proved from what axiom?

38. An exterior angle of a triangle is greater than . . . what?

39. If two angles of a triangle are unequal, . . . what?

40. State six methods of proving lines equal.

41. State six new methods of proving angles equal.

42. State a method of proving that two straight lines lie in one straight line.

43. What is the method of drawing an angle equal to a given angle?

44. The statement of a theorem is usually in what form?

45. If a theorem is true, is its converse necessarily true? Its opposite?

46. If a theorem is true, is the converse of the opposite true?

47. If a line is to be proved perpendicular to a given line, what equality may be proved?

48. In a proof, a statement is often obtained almost like the one desired; but one term may need to be changed. How is it possible to make this change in an equation or inequality?

49. In an inequality, give another method of making this change.

50. If two lines are in the same triangle, and are to be proved unequal, what two methods are probable?

51. In different triangles, what one method?

52. How is an exterior angle of a triangle formed?

53. How many can there be?

MISCELLANEOUS EXERCISES

1. If at the ends of a straight line perpendiculars to it are drawn, these perpendiculars cut off equal parts upon any oblique line drawn through the middle point of the given line.

2. If two triangles are congruent, the bisector of an angle of the one equals the bisector of the corresponding angle of the other.

3. If equal distances are measured on the sides of an angle from its vertex, the points thus obtained are equally distant from any point in the bisector of the angle.

4. If on the arms of an isosceles triangle equal distances are measured from the vertex, the lines joining these points to the opposite ends of the base are equal.

5. If the arms of an isosceles triangle are extended the same length through the vertex, the lines joining their ends to the ends of the base are equal.

6. If on the arms of an isosceles triangle equal distances are measured from the ends of the base, these points are equally distant from the opposite ends of the base.

7. If on the base of an isosceles triangle equal distances are measured from the ends of the base, the lines joining these points to the vertex are equal.

8. If the base of an isosceles triangle is extended both ways the same distance, the lines joining the ends to the vertex are equal.

9. The angle formed by two lines drawn within a triangle from the ends of one side is greater than the opposite angle.

10. The sum of the altitudes of a triangle is less than the perimeter.

11. In triangle ABC, AB is greater than AC, and the bisectors of angles B and C meet at O. Prove $BO > CO$.

12. In a quadrilateral $ABCD$, $AB = CD$, and $\angle C > \angle B$; prove diagonal BD greater than diagonal AC.

13. The line through the intersection of the bisectors of the base angles of a triangle and parallel to the base cuts off, on the sides, parts whose sum equals the parallel line.

14. If from a point outside an angle or its vertical angle perpendiculars are drawn to the sides of the angle, an angle is formed which is equal to the given angle.

15. The perpendicular from the vertex of the right angle of a right triangle to the hypotenuse divides the right angle into two parts equal respectively to the other angles of the triangle.

16. If a perpendicular is drawn from the middle point of the base of an isosceles triangle to one of the arms, this line makes with the base an angle equal to one-half the vertex angle.

17. Find each angle of a triangle if the first angle equals twice the second and the third equals three times the first.

18. If one angle of a triangle is $x°$, another angle $y°$, what expression represents the third angle?

19. If the vertex angle of an isosceles triangle is $x°$, what expression represents either base angle?

20. Trisect a right angle.

21. A line through the intersection of the diagonals of a parallelogram and ending in the sides is bisected by the intersecting point.

22. If each half of each diagonal of a parallelogram is bisected, the lines joining these points form parallelogram.

23. If the upper base of a trapezoid is one-half the lower base, the upper part of each diagonal is one-half the lower part.

24. The angle formed by the bisectors of the base angles of an isosceles triangle is equal to the exterior angle at one end of the base.

25. The diagonals of a rhombus divide the figure into four congruent triangles.

26. If a line is drawn across an angle perpendicular to the bisector of the angle, two congruent triangles are formed.

27. If perpendiculars are drawn from the ends of the base of a parallelogram, and the upper base is extended, two congruent right triangles are formed.

28. If two adjacent sides of a quadrilateral are equal and the diagonal bisects their included angle, the other two sides are equal.

29. In an isosceles triangle the vertex angle is 50°. Find the base angles.

30. How many degrees are there in each angle of an equiangular 9-gon? In each exterior angle?

31. If one acute angle of a right triangle is double the other, how many degrees in each?

32. If one acute angle of a right triangle is four times the other, how many degrees in each?

33. If the vertex angle of an isosceles triangle equals twice the sum of the base angles, how many degrees in each angle?

34. If one angle of a parallelogram is 40°, find each angle.

35. The exterior angle at the base of an isosceles triangle equals one-half the vertex angle plus a right angle.

36. If from any point in the base of an isosceles triangle, perpendiculars to the equal sides are drawn, they make equal angles with the base.

37. The bisectors of the angles of a trapezoid form a quadrilateral, two of whose angles are right angles.

38. The bisectors of the angles of a rectangle form a square.

39. If from any point in the base of an isosceles triangle parallels to the equal sides are drawn, the perimeter of the parallelogram equals the sum of the equal sides of the triangle.

40. The lines joining in order the middle points of the sides of a rectangle form a rhombus.

41. The bisectors of two exterior angles of a triangle and of the third interior angle meet in a point.

42. If AD is the bisector of $\angle A$ of $\triangle ABC$, and BE is $\parallel$ to AD and meets AC extended at E, prove ABE an isosceles $\triangle$.

43. If AD is the bisector of exterior $\angle FAB$ of $\triangle ABC$, and BE is $\parallel$ to AD and meets AC at E, prove $\triangle ABE$ isosceles.

44. If two adjacent angles of a quadrilateral are right angles, the bisectors of the other angles are perpendicular.

45. If two opposite angles of a quadrilateral are right angles, the bisectors of the other angles are parallel.

46. The line joining the middle points of the diagonals of a trapezoid equals one-half the difference of the bases.

47. If two triangles are congruent, the corresponding medians are equal.

48. If two triangles are congruent, the corresponding altitudes are equal.

49. If one diagonal of a quadrilateral divides it into isosceles triangles, the other diagonal bisects two of the angles and is the perpendicular bisector of the first diagonal.

50. The bisectors of two exterior angles of a triangle form an angle equal to one-half the sum of the adjacent interior angles.

51. How many sides has an equiangular polygon, if an exterior angle equals 36°?

52. How many degrees in an interior angle of an equiangular polygon of 16 sides?

53. Two rectangles are congruent if they have two adjacent sides respectively equal.

54. If two isosceles triangles have the same base, the line joining their vertices bisects the angles.

55. If a quadrilateral has two opposite pairs of adjacent sides equal, the angle-bisectors meet in a point equally distant from the sides.

CIRCLES

CHAPTER XI

CIRCLES AND TANGENTS

The basic principle is the definition:

A circle is a plane figure bounded by a curved line, all points of which are equally distant from a point within called the center.

157. Define **Secant**; also **Circle**; **Radius**; **Arc.**

158. Define **Chord.**

159. Define **Diameter.**

160. a. *Only one circle can be drawn with a given center and a given radius.*

b. *All radii of the same or equal circles are equal.*

c. *A diameter equals two radii.*

d. *All diameters of the same or equal circles are equal.*

e. *Two circles are equal if they have equal radii.*

f. *A diameter bisects the circumference.*

161. Define **Tangent.** 162. Define **Segment.**

163. Define **Sector.** 164. Define **Central Angle.**

165. Define **Circumscribed.** 166. Define **Inscribed.**

*167. *Through three points not in a straight line, one circle and only one can be drawn.*

168. *Through three (or more) points in a straight line, a circle cannot be drawn.*

*169. PROBLEM: *Circumscribe a circle about a triangle.*

*170. *A straight line perpendicular to a radius at its end is a tangent to the circle.*

*171. *A tangent to a circle is perpendicular to the radius drawn to the point of tangency.*

*172. *Two tangents to a circle from an external point are equal.*

*173. PROBLEM: *Inscribe a circle in a triangle.*

*174. *A circle can be circumscribed about or inscribed in any regular polygon.*

SUMMARY

175. State two methods of proving a line tangent to a circle.

State one additional method of proving lines perpendicular.

176. EXERCISES

1. Draw a tangent to a circle at a point on the circumference.

2. A perpendicular to a tangent at the point of tangency passes through the center.

3. If two circles are tangent, the two radii to the point of tangency are in a straight line.

4. If a line is drawn from the center of a circle to the intersection of two tangents, this line: *a.* Bisects the angle between the tangents; *b.* Is the perpendicular bisector of the chord between the points of tangency.

5. If two circles are tangent externally, the tangents to them from any point in the common internal tangent are equal.

6. The sum of two opposite sides of a circumscribed quadrilateral is equal to the sum of the other two sides.

7. If two common external tangents are drawn to two circles, the parts intercepted between the points of tangency are equal.

8. Find the locus of the centers of circles, which have a given radius and touch a given line.

9. Find the locus of the centers of circles, which touch a given line at a given point.

10. Find the locus of the centers of circles, which touch each of two parallel lines.

11. Find the locus of the centers of circles, which touch each of two non-parallel lines.

12. Find the locus of points at a given distance from a given point.

13. Find a point at a given distance from a given point, and equidistant from two given points.

14. Find a point at a given distance from a given point, and equidistant from two non-parallel lines.

15. Find a point at a given distance from a given point, and equidistant from two parallel lines.

16. Find a point at a given distance from a given point, and at a given distance from a given line.

CHAPTER XII

EQUALITY OF ARCS AND CHORDS

The two basic methods are:

177. In the same or equal circles, equal central angles intercept equal arcs.

178. In the same or equal circles, equal arcs are intercepted by equal central angles.

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*179. *In the same or equal circles, equal chords subtend equal arcs.*

*180. *In the same or equal circles, equal arcs are subtended by equal chords.*

*181. *A radius perpendicular to a chord of a circle bisects the chord and its arc.*

*182. *Parallel lines intercept equal arcs on a circumference.* (Three cases.)

*183. a. *In the same or equal circles, equal chords are equally distant from the center.*

b. *In the same or equal circles, chords equally distant from the center are equal.*

184. PROBLEM: *Given an arc, find the center of the circle.*

185. PROBLEM: *Bisect a given arc.*

SUMMARY

186. State four methods of proving arcs equal.

State four additional methods of proving straight lines equal.

187. EXERCISES

1. Prove Art. 160f. by Art. 177.

2. There are four ideas in Art. 181: (1) Radius; (2) perpendicular; (3) bisects the chord; (4) bisects the arc. If any two of these are given, the other two can be proved. In Art. 181 (1) and (2) were given, and (3) and (4) were proved.

Let (1) and (3) be given; prove (2).

Make three other combinations and prove them.

3. If two circles intersect, the line of centers is the perpendicular bisector of the common chord.

4. If A, B, D, and C are four points in order on a circumference such that BC equals AD, then chord AB equals chord CD.

5. If A, B, D, and C are four points in order on a circumference, such that AB equals CD, then chord BC equals chord AD.

6. The opposite arcs intercepted by two diameters are equal.

7. Two chords drawn perpendicular to a third chord at its ends are equal.

8. Two parallel chords drawn through the ends of a diameter are equal.

9. If two unequal circles have the same center (concentric), any two chords of the larger circle tangent to the smaller circle are equal.

10. If from a point in a circumference two chords are drawn making equal angles with the radius to that point, these chords are equal.

11. If two intersecting chords make equal angles with the radius through the point of intersection, these chords are equal.

12. The diameter of the circle inscribed in a right triangle equals the difference between the sum of the short sides and the hypotenuse.

13. Find the locus of the middle points of chords of a given length in a given circle.

14. Find the locus of the middle points of chords drawn from a given point in a given circumference.

15. Find the locus of the middle points of straight lines drawn from a given external point to a given circumference.

16. Find the locus of the end points of straight lines of a given length, touching a given circumference at one end and parallel to a given line.

17. A ladder, leaning against a perpendicular wall, slips down until it is flat upon the ground. Find the locus of its middle point.

18. Divide a circle into four equal arcs; six; three.

19. A trapezoid inscribed in a circle is isosceles.

20. If two circles are tangent to two parallel lines, any other parallel line, cutting the circle, forms two equal chords.

CHAPTER XIII

INEQUALITY OF ARCS AND CHORDS

The basic method is:
The whole is greater than any one of its parts.

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188. In the same or equal circles, the greater of two central angles intercepts the greater arc.

189. *In the same or equal circles, the greater of two arcs is intercepted by the greater central angle.*

***190.** *In the same or equal circles, the greater of two chords subtends the greater arc.*

191. *In the same or equal circles, the greater of two arcs is subtended by the greater chord.*

***192.** *In the same or equal circles, the greater of two chords is nearer the center.*

***193.** *In the same or equal circles, if two chords are unequally distant from the center, the chord nearer the center is the greater.*

SUMMARY

194. State two methods of proving arcs unequal.

195. State two additional methods of proving straight lines unequal.

196. EXERCISES

1. The diameter of a circle is greater than any other chord. (Two ways.)

2. Prove Art. 193 by the Indirect Method.

3. A chord drawn perpendicular to a diameter is smaller than any other chord through the point of intersection.

4. If *AB* and *CD* are two intersecting chords of a circle, and *AB* is greater than *CD*, and arcs *ACB* and *DAC* are each less than a semicircumference, *BC* is greater than *AD*.

5. In the same figure, if *BC* is greater than *AD*, prove *AB* greater than *CD*.

6. If two chords are drawn from a point in a circumference, the chord making the smaller angle with the radius to that point is the larger chord.

7. The perpendicular from the center of a circle to the side of an inscribed equilateral triangle is less than the perpendicular from the center to the side of an inscribed square.

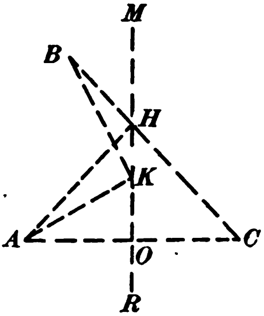
8. If point E be taken on side AB of isosceles triangle ABC, prove EC greater than EB. $(AB = AC.)$

9. In triangle ABC, AB is greater than AC, and the median AE is drawn. Prove the angle AEB greater than angle AEC.

10. A method of laying out the shortest possible railroads from two towns to a common junction on the main line:

Let MR be the main line, A and B the two towns. Draw $AO \perp MR$, and extend it its own length to C. Draw BC intersecting MR at H,

$BH + AH < BK + AK$ or any other combination.



11. In a given line, find a point at a given distance from a given point.

12. Find a point at a given distance from two given points.

13. Find the locus of points at a given distance from a given circumference.

14. Find the locus of the centers of circles which have a given radius and pass through a given point.

15. Find the locus of the centers of circles which pass through two given points.

16. Construct a circle which has a given radius and passes through two given points.

17. Construct a circle which has its center in a given line and passes through two given points.

CHAPTER XIV

MEASUREMENT OF ANGLES

The basic method is:

197. **A central angle has the same measure as its intercepted arc.**

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198. Define **Measurement; Unit of Measure.**

199. Define **Numerical Measure.** 200. **Degree.**

201. What is the unit of measure for angles?

202. Define **Commensurable.** 203. **Incommensurable.**

*204. **An inscribed angle has the same measure as one-half its intercepted arc.** (Three cases.)

205. *Angles inscribed in the same or equal segments of a circle are equal..*

206. *An angle inscribed in a semicircle is a right angle.*

207. *The opposite angles of an inscribed quadrilateral are supplementary.*

*208. *An angle between a tangent and a chord, meeting on the circumference, has the same measure as one-half the intercepted arc.*

*209. *An angle between two chords meeting within the circle has the same measure as one-half the sum of the intercepted arcs.*

*210. *An angle between a tangent and a secant, two tangents, or two secants, meeting without the circle, has the same measure as one-half the difference of the intercepted arcs.* (Three cases.)

211. Group 197, 204, 208, 209, and 210 under one statement.

*212. PROBLEM: *Erect a perpendicular at the end of a line.*

*213. PROBLEM: *Draw a tangent to a circle from a point on the circumference.*

*214. PROBLEM: *Draw a tangent to a circle from a point without the circle.*

*215. PROBLEM: *Upon a given straight line describe a segment of a circle in which any inscribed angle will equal a given angle.*

SUMMARY

216. State the measure of each of the following angles:

a. At the center of a circle.

b. Between the center and the circumference.

c. At the circumference, whose sides are chords or tangents.

d. Outside the circle, whose sides are secants or tangents.

217. State two additional methods of proving angles equal.

218. State one additional method of proving lines perpendicular.

219. EXERCISES

1. Prove Art. 208 by drawing a radius perpendicular to the chord and a radius to the point of tangency. Prove, also, by drawing a line from the end of the chord parallel to the tangent.

2. Prove Art. 209 by drawing a line from the end of one chord parallel to the other.

3. Prove Art. 210 in the same way.

4. Prove ex. 6, Art. 187, by measurement of angles.

5. Prove ex. 7, Art. 187, by measurement of angles.

6. Prove ex. 9, Art. 187, by measurement of angles.

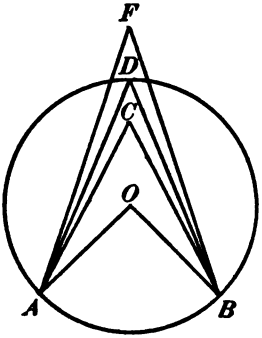
7. If the bisector of an inscribed angle is extended to the circumference and through this point a chord is drawn parallel to one side of the angle, this chord equals the other side of the angle.

8. An exterior angle of an inscribed quadrilateral equals the opposite interior angle.

9. If two circles are tangent, a line through the point of tangency subtends equal (in degrees) arcs.

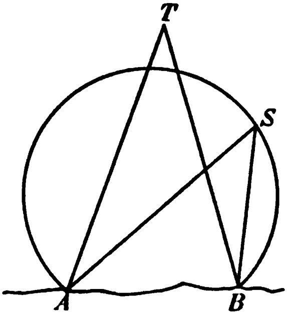
10. If two circles are tangent externally, and a line is drawn through the point of tangency and ending in the circumference, tangents at the ends of this line are parallel.

11. If two circles are tangent externally, and two lines are drawn through the points of tangency and ending in the circumference, the chords joining the ends of these lines are parallel.



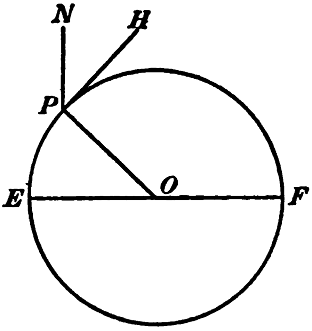
12. If two circles intersect, and a line is drawn through each point of intersection and ending in the circumferences, the chords joining the ends of these lines are parallel.

13. In the figure, prove $\angle AOB > \angle ACB$; $\angle ACB > \angle ADB$; $\angle ADB > \angle AEB$.



14. An application of ex. 13 in determining the "angle of safety" in navigation:

A certain section of a bay is dangerous on account of hidden rocks and shoals. To enable sailors to keep out of this section both by day and by night, two lighthouses, A and B, are built upon the shore. Then on a map a circle is drawn through A and B, and enclosing the dangerous area. An inscribed angle ASB is measured. This is the angle of safety; and if the angle from any boat T to A and B is less than the angle of safety, the boat is in safe waters.



15. A method of finding the latitude of a point on the earth north of the equator:

Let O be the center of the earth; EF the equator; P the point; PH a line to the horizon; PN a line to the north star. The measure of $\angle HPN$ is the latitude of P.

Proof: $\angle POE$ is measured by PE.

$\angle HPN = \angle POE$.

$\angle HPN$ is measured by PE.

As PE is the latitude of P, the measure of $\angle HPN$ is the latitude of P.

CHAPTER XV

REGULAR POLYGONS

The basic method is:

A regular polygon is equilateral and equiangular. (Art. 46b.)

***220.** *If the circumference of a circle is divided into any number of equal arcs,* (1) *the chords to the points of division form a regular inscribed polygon;* (2) *the tangents at the points of division form a regular circumscribed polygon.*

221. *If the arcs subtended by the sides of a regular inscribed polygon are bisected, the tangents at the points of bisection form a regular circumscribed polygon of the same number of sides.*

222. What is the **Radius** of a regular polygon? What can be said about all radii of a regular polygon? What is the **Apothem** of a regular polygon? What can be said about all apothems of a regular polygon?

223. What is the **Angle at the Center** of a regular polygon? How many central angles are there? What can be said about all angles at the center of a regular polygon?

224. Each angle at the center equals 360° divided by . . . what?

***225.** *If two diameters of a circle are perpendicular, the chords joining their ends form a square.*

***226.** PROBLEM: *Inscribe a regular hexagon in a circle.*

***227.** PROBLEM: a. *Given a regular inscribed polygon, construct a regular inscribed polygon of double the number of sides.* NOTE. Bisect the arcs.

b. *Given a regular circumscribed polygon, construct a regular circumscribed polygon of double the number of sides.*

228. Define **Perimeter.**

229. For discussion:

a. *The circumference of a circle lies between the perimeters of the inscribed and circumscribed regular polygons.*

b. *The area of a circle lies between the areas of the inscribed and circumscribed regular polygons.*

c. *Of two regular polygons inscribed in a circle, the one with the greater number of sides is more nearly equal to the circle, and its perimeter is more nearly equal to the circumference.*

d. *Of two regular polygons circumscribed about a circle, the one with the greater number of sides is more nearly equal to the circle, and its perimeter is more nearly equal to the circumference.*

e. *If the number of sides of a regular inscribed or circumscribed polygon is indefinitely increased:*

1. *The perimeter of the polygon is said to approach the circumference as its limit.*
2. *The area of the polygon is said to approach the area of the circle as its limit.*

230. PROBLEM: *If the radius of a circle is* 1, *the circumference is* > 6, *and* < 8.

SUMMARY

231. State two methods of proving polygons regular.

232. EXERCISES

1. An equilateral polygon inscribed in a circle is regular.
2. An equiangular polygon circumscribed about a circle is regular.
3. Circumscribe a square about a circle.
4. Circumscribe a regular hexagon about a circle.

5. Inscribe an equilateral triangle; circumscribe an equilateral triangle.

6. Inscribe and circumscribe regular polygons of eight sides; sixteen sides; twelve sides.

7. A central angle of a regular polygon $= \frac{360°}{n}$.

8. An interior angle of a regular polygon $= \frac{2n-4}{n} \times 90°$.

9. The central angles of two regular polygons, of the same number of sides, are equal.

10. Find the number of degrees in the angle at the center of a regular octagon; of a regular polygon of ninety-six sides.

11. Find the number of degrees in an interior angle of a regular dodecagon; of a regular polygon of sixty-four sides.

12. How many sides has a regular polygon, whose angle at the center is 30°?

13. The radius drawn to any vertex of a regular polygon bisects the angle at the vertex.

14. The apothem of an inscribed equilateral triangle equals one-half the radius.

15. On a given straight line as a side, construct a square; regular octagon; regular hexagon; regular dodecagon.

233. CONSTRUCTION PROBLEMS

Construct one and only one triangle, having given:

1. Two sides and the included angle. (Art. 81.)

2. Two angles and the included side. (Art. 82.)

3. Three sides. (Art. 66.)

4. Two angles and an opposite side. (Find the third angle.)

5. Two sides and an opposite angle, provided that the opposite side is greater than the other given side.

Which of the previous five cases is each of the following cases of constructing one and only one right triangle, having given:

6. Two arms?

7. Arm and adjacent acute angle?

8. Arm and opposite acute angle?

9. Hypotenuse and acute angle?

10. Hypotenuse and arm?

Let the base angles of an isosceles triangle be B and C, the opposite sides b and c respectively, the base a, and vertex angle A:

Construct an isosceles triangle, having given:

11. b and $\angle B$.

12. b and $\angle A$.

13. a and $\angle C$.

14. a and $\angle A$.

15. a and b.

234. State four cases of constructing one and only one triangle.

235. State the four steps in the analysis of construction problems.

NOTE. What is the custom in naming the lines and angles of a triangle?

236. EXERCISES

Construct a triangle, having given

1. a, b, m_b.
2. $a, m_a, \angle B$.
3. b, c, h_a.
4. $a, \angle B, h_a$.
5. $a, \angle A, m_a$.
6. $a, \angle B, m_c$.
7. $b, \angle B, h_b$.
8. $c, \angle A, a + b$.
9. $b, c, \angle B$; $(b < c)$.

REVIEW QUESTIONS

1. Define Circle; Circumference; Radius; Arc.

2. Define Secant; Chord; Diameter.

3. Define Tangent; Central Angle; Inscribed Angle.

4. Define Segment; Sector; Inscribed Circle; Circumscribed Circle.

5. When are two circles equal?

6. Are two circles congruent if they are equal?

7. State two methods of proving a line tangent to a circle.

8. There are now eight methods of proving lines perpendicular. Name them.

9. How many points determine a straight line?

10. How many points determine a circle?

11. Can a circle be drawn through three points in a straight line?

12. Equality of arcs, being a new subject, is first proved by what fundamental principle of equality?

13. Chords, being straight lines, are proved equal by what method?

14. State four methods of proving arcs equal.

15. There are now ten methods of proving lines equal. Name most of them.

16. Inequality of arcs, being a new subject, depends upon what axiom?

17. Inequality of chords will be proved by the methods of what class?

18. State two methods of proving arcs unequal.

19. There are now nine methods of proving lines unequal. Name most of them.

20. Define Measurement; Numerical Measure; Unit of Measure.

21. What is a Degree?

22. What is a unit of measure for angles?

23. What is meant by Commensurable and Incommensurable quantities?

24. What is the basic theorem for measurement of angles?

25. What is the most useful theorem?

26. What can be said of the opposite angles of an inscribed quadrilateral?

27. Group 197, 204, 208, 209, and 210 under one statement.

28. An angle at the center of a circle is measured by what?

29. An angle between the center and the circumference is measured by what?

30. An angle at the circumference whose sides are chords or tangents is measured by what?

31. An angle outside a circle whose sides are secants or tangents is measured by what?

32. State two new methods of proving angles equal.

33. Define Regular Polygon; its Radius; its Apothem; its Angle at the center.

34. Each angle at the center of a regular polygon of *n* sides equals what?

35. Each interior angle equals what?

36. The side of a regular inscribed hexagon equals what?

37. A regular triangle is called what? A regular quadrilateral?

38. Define Perimeter.

39. The circumference of a circle lies between what?

40. If the number of sides of a regular inscribed or circumscribed polygon is indefinitely increased, how does the polygon change?

41. State two methods of proving polygons regular.

42. What is a method of inscribing a regular polygon in a circle?

43. What is meant by: A central angle is measured by its intercepted arc?

44. The apothem of a regular circumscribed hexagon equals what?

45. The side of a circumcribed square equals what?

46. The diagonal of an inscribed square equals what?

47. State four steps in the analysis of construction problems.

48. What is the custom in naming the sides and angles of a triangle?

49. How is distance measured from a point to a straight line? From a point to a circle?

50. What can be said about equal chords in a circle?

MISCELLANEOUS EXERCISES

1. The perpendicular bisectors of the sides of an inscribed polygon meet at a common point.

2. The perpendiculars to the sides of circumscribed polygon at the points of tangency meet at a common point.

3. The bisector of the angle between two tangents to a circle passes through the center.

4. The bisectors of the angles of a circumscribed polygon meet at a common point.

5. An inscribed trapezoid is isosceles.

6. A chord is parallel to the tangent at the middle point of the subtended arc.

7. A circumscribed parallelogram is equilateral.

8. If two circles are concentric and a common secant is drawn, the parts of the secant between the circumferences are equal.

9. If two equal secants are drawn from a point to a circle, the parts within the circle are equal.

10. If two equal secants are drawn from a point to a circle, they are equally distant from the center.

11. The radius of the circle inscribed in an equilateral triangle is one-half the radius of the circumscribed circle.

12. If chord BA of a circle is produced to C making AC equal the radius, and COE is drawn through the center, $\angle BOE = 3 \angle C$.

13. If perpendiculars are drawn from the ends of a diameter to a tangent, the sum of the perpendiculars will equal the diameter.

14. If two circles are tangent externally, the common interior tangent bisects their common exterior tangent.

15. The common interior tangents of two circles are equal.

16. The line bisecting an inscribed angle bisects the intercepted arc.

17. The line joining the vertex of an inscribed angle to the middle point of its intercepted arc bisects the angle.

18. The sum of the interior angles of a triangle equals two right angles. (Prove by circumscribing a circle and drawing a tangent at one vertex.)

19. The circle drawn on one of the sides of an isosceles triangle as a diameter bisects the base.

20. An inscribed parallelogram is a rectangle.

21. The diagonals of a rectangle inscribed in a circle are diameters.

22. A circumference drawn on the hypotenuse of a right triangle as a diameter passes through the vertices of all right triangles having the same hypotenuse.

23. If two circles are tangent externally at C, and a common external tangent touches them at A and B, the angle ACB is a right angle. (Draw the internal tangent.)

24. A circle drawn on the radius of another circle as a diameter bisects all chords of the larger circle drawn from the point of tangency.

25. If a circle is drawn upon one arm of a right angle triangle as a diameter, and a tangent is drawn at the point of its intersection with the hypotenuse, this tangent bisects the other arm.

26. If two equal circles are tangent and a straight line is drawn through the point of tangency and ending in the circumferences, the parts of the line are equal.

27. What is the locus of all points from which two equal tangents can be drawn to two tangent circles?

28. What is the locus of the vertices of the triangles which have a given acute angle and a given side opposite?

29. Inscribe a circle in a sector.

30. Inscribe a circle in a quadrilateral having two opposite pairs of adjacent sides equal.

31. The line of centers of two equal circles bisects their common internal tangent.

32. If semicircles are drawn on the sides of an equilaterial triangle, these arcs intersect at the middle points of the sides of the triangle.

33. Find the locus of the points of intersection of the diagonals of trapezoids on the same base and with the same base angles.

34. Two circles intersect at C and D. Diameters CA and CB are drawn, and chords AD and BD. Prove ADB a straight line.

35. In an equilateral triangle construct three equal circles each tangent to two sides and to two circles.

36. In a given circle construct three equal circles each tangent to the given circle and to two others.

37. Through a given point on the bisector of an angle construct a circle tangent to the sides of the angle.

38. Construct a circle passing through a given point and tangent to a given line at a given point.

39. Two equal circles are tangent externally. Find the locus of the centers of circles tangent to both given circles externally.

40. By means of a circle construct a right triangle, having given the hypotenuse and an arm.

41. By means of a circle construct a right triangle, having given the hypotenuse and an acute angle.

42. Also, having given the hypotenuse and the altitude upon the hypotenuse.

43. If the angle between two tangents to a circle is 40°, how many degrees in each intercepted arc?

44. Through two given points draw parallel lines at a given distance apart.

45. Construct an isosceles right triangle having given the hypotenuse.

46. Construct a triangle ABC, given $AB = 2''$ (inches), $\angle B = 75°$, and the median to $AB = 2\frac{1}{4}''$.

47. Construct a right triangle, having given an acute angle and the sum of the arms.

48. Construct an isosceles triangle, having given the perimeter and the altitude.

49. Construct a triangle, having given one side, an adjacent angle, and the sum of the other two sides.

50. Construct an equilateral triangle, having given (*a*) the perimeter; (*b*) the altitude; (*c*) the median; (*d*) the radius of the inscribed circle; (*c*) the radius of the circumscribed circle.

51. Construct a square, having given the sum of the diagonal and a side; having given a diagonal.

52. Construct a triangle having given A, B, and the radius of the circumscribed circle.

53. Construct a regular dodecagon, having given a side.

54. Construct a common external tangent to two unequal circles.

55. Construct a common internal tangent to two unequal circles.

Construct a triangle having given:

56. a, h_a, m_a.

57. $h_a, h_b, \angle C$.

58. $a, \angle A, h_a$.

59. a, b, h_b.

60. $m_a, h_a, \angle B$.

61. $a, b, \angle A + \angle B$.

Construct a rectangle having given:

62. A side and a diagonal.

63. A side and the angle formed by the diagonals.

64. The perimeter and a diagonal.

65. A diagonal and the angle formed by the diagonals.

Construct a rhombus having given:

66. A side and a diagonal.

67. The perimeter and a diagonal.

68. One angle and a diagonal.

69. The two diagonals.

Construct a parallelogram, having given:

70. Two non-parallel sides and an angle.

71. One side and the diagonals.

72. One side, an angle, and the altitude to that side.

73. The diagonals and the angle between them.

74. Two non-parallel sides and the altitude.

75. Two non-parallel sides and a diagonal.

A trapezoid can always be divided into a parallelogram and a triangle. Therefore, in constructing a trapezoid, construct the parallelogram and the triangle. Construct a trapezoid having given:

76. The bases and a diagonal. (Isosceles.)

77. Two sides and their included angle. (Isosceles.)

78. The bases and the angles adjacent to one base.

79. The bases, the altitude, and an angle.

80. The bases and the altitude. (Isosceles.)

Construct a circle that will:

81. Touch a given circle at P, and pass through a point Q.

82. Pass through point P, and touch two given parallels.

83. Touch two given lines, one of them at a point P.

84. Touch three given non-parallel lines.

85. Touch two given circles and have a given radius.

86. Construct a triangle, having given the mid-points of the three sides.

PROPORTION

CHAPTER XVI

PROPORTION FROM PARALLEL LINES

The basic method is:

*237. **Three or more parallel lines intercept proportional parts on two transversals.**

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238. Define **Ratio.**

239. Define **Proportion.**

240. Define **Extremes** and **Means.**

Define **Antecedents and Consequents.**

241. Define **Alternation, Inversion, Composition, and Division** in proportion.

242. The product of the means equals what?

243. What is the converse of Art. 242?

244. If three terms of one proportion are equal respectively to three terms of another proportion, what can be said of the remaining terms?

245. **A straight line parallel to one side of a triangle divides the other two sides proportionally.**

246. Arrange the proportion of 245 by composition.

*247. PROBLEM: *Find the fourth proportional to three given straight lines.*

248. PROBLEM: *Divide a given straight line into parts proportional to any number of given straight lines.*

*249. *The bisector of an angle of a triangle divides the opposite side into parts proportional to the other two sides.*

250. Explain **Internal division** of a line. **External division.**

*251. *The bisector of an exterior angle of a triangle divides the opposite side, externally, into parts proportional to the other two sides.*

*252. *A line dividing two sides of a triangle proportionally is parallel to the third side.*

253. Define **Similar Polygons.**

SUMMARY

254. State two methods of proving lines in proportion.

255. State a new method of proving lines parallel.

256. EXERCISES

1. If a line is parallel to two sides of a parallelogram and intersects the other two sides, it divides these sides proportionally.

2. If a line is parallel to the bases of a trapezoid and intersects the other two sides, it divides these sides proportionally.

3. Prove Art. 106, ex. 13, by Art. 252; also ex. 14, 15, and 16.

4. Prove Art. 116 by Art. 252.

5. If a line divides two opposite sides of a parallelogram proportionally, it is parallel to the other sides.

6. If a line divides the non-parallel sides of a trapezoid proportionally, it is parallel to the bases.

7. If a, b, and c are the first three terms of a proportion, show that the fourth proportional $x = \frac{bc}{a}$.

8. Find the fourth proportional to 2, 3, and 4, arithmetically and geometrically.

9. Find the fourth proportional to 3, 5, and 2, arithmetically and geometrically.

10. In triangle ABC, $AB = 6$; $AC = 5$; a line $EF \parallel BC$ divides AC into parts; $AF = 4$ and $FC = 1$. Find AE and EB.

11. In $\triangle ABC$, $EF \parallel BC$; $AF = 2EB$; $AE = 20$; $FC = 10$. Find AF.

12. In triangle ABC, $AB = 12$; $AC = 14$; $BC = 13$. Find the parts of BC made by the bisector of angle A.

13. In the same figure, find the parts of AC made by the bisector of angle B; also of AB similarly.

CHAPTER XVII

SIMILAR TRIANGLES

The basic method is:

***257. Two triangles are similar if two angles of the one are equal respectively to two angles of the other.**

***258. Two triangles are similar if an angle of the one equals an angle of the other and the including sides are in proportion.**

259. What can be said of the corresponding sides of similar triangles? The corresponding angles?

260. What are the basic theorems of similar triangles? How are the other theorems proved?

261. *Two right triangles are similar if an acute angle of the one equals an acute angle of the other.*

262. *Two right triangles are similar if the two arms of the one are in proportion with the two arms of the other.*

263. *Two right triangles are similar if the hypotenuse and arm of the one are in proportion with the hypotenuse and arm of the other.*

264. Two triangles are similar if their corresponding sides are in proportion.

265. From Articles 261, 262, and 263, a general theorem for all right triangles can be made:

Two right triangles are similar if an acute angle of the one equals an acute angle of the other, or if two sides of the

one are in proportion with two corresponding sides of the other.

***266.** PROBLEM: *On a given straight line construct a triangle similar to a given triangle.*

SUMMARY

267. State four methods of proving triangles similar, covering all cases.

What is the principal use of similar triangles? How can the corresponding sides be found?

268. EXERCISES

1. If two triangles are mutually equiangular, they are similar.

2. The line crossing two sides of a triangle and parallel to the third side cuts off a triangle similar to the given triangle.

3. Two isosceles triangles are similar if an angle of the one equals a corresponding angle of the other.

4. If E and H are points on side AB of $\angle ABC$, and EF and HK are $\perp BC$, $\triangle EBF \backsim \triangle HBK$.

5. If two chords AB and CD intersect at E within a circle, $\triangle AEC \backsim \triangle BED$.

6. If two chords AB and CD intersect at E without a circle, when extended, $\triangle AED \backsim \triangle BEC$.

7. Two triangles are similar if their sides are respectively parallel each to each.

8. Two triangles are similar if their sides are respectively perpendicular each to each.

9. Two circles are tangent externally at H, and through H three lines are drawn meeting one circle in A, B, and C, and the other circle in D, E, and F. Prove $\triangle ABC \backsim \triangle DEF$.

10. If BD and CE are altitudes of $\triangle ABC$, $\triangle ABD \backsim \triangle ACE$.

11. In the same figure, if BD and CE intersect at O, $\triangle BOE \backsim \triangle COD$.

12. In the same figure, prove $\triangle ABC \backsim \triangle AED$.

13. The lines joining the middle points of the sides of a triangle form a triangle similar to the given triangle.

CHAPTER XVIII

PROPORTION FROM SIMILAR TRIANGLES

The basic method is:

The corresponding sides of similar triangles are in proportion. (Art. 259.)

***269.** *The corresponding altitudes of two similar triangles are in proportion with any two corresponding sides.*

270. State the method of proving that the product of two quantities equals the product of two other quantities.

***271.** *If two chords intersect within a circle, the product of the parts of one chord equals the product of the parts of the other chord.*

***272.** *If two secants intersect without a circle, the product of one whole secant and its external part equals the product of the other whole secant and its external part.*

273. *If a secant and a tangent intersect without a circle, the product of the whole secant and its external part equals the square of the tangent.*

274. Group 271, 272, and 273 under one statement.

275. Define **Mean Proportional. Third Proportional.**

***276. If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse:**

1. **The triangles formed are similar to the given triangle and to each other.**

2. *The perpendicular is the mean proportional between the parts of the hypotenuse.*

3. *Either arm is the mean proportional between the whole hypotenuse and its adjacent part.*

277. *The perpendicular from any point in the circumference of a circle to the diameter is a mean proportional between the parts of the diameter.*

***278.** PROBLEM: *Construct a mean proportional between two given straight lines.*

SUMMARY

279. State two new methods of proving lines in proportion. Recall the other two.

280. State three methods of proving a line a mean proportional between two other lines.

281. EXERCISES

1. The corresponding medians of two similar triangles are in proportion with any two corresponding sides.

2. The corresponding angle-bisectors of two similar triangles are in proportion with any two corresponding sides.

3. The diagonals of a trapezoid divide each other proportionally.

4. The radii of the circles inscribed in two similar triangles are in proportion with any two corresponding sides.

5. The radii of the circles circumscribed about two similar triangles are in proportion with any two corresponding sides.

6. In any right triangle, the product of the hypotenuse and the altitude upon it equals the product of the arms.

7. The product of any altitude of a triangle and its corresponding side equals the product of any other altitude and its corresponding side.

8. Find a mean proportional between two lines a and b by three different methods.

9. If a and c are the extremes of a proportion, show that the mean proportional $x = \sqrt{ac}$.

10. Find a mean proportional between 1 and 4, arithmetically and geometrically.

11. If x is a third proportional to a and b, show that $x = \frac{b^2}{a}$.

12. Find a third proportional between lines a and b.

13. Find a third proportional to 2 and 3, arithmetically and geometrically.

14. Find geometrically $\sqrt{3}$; $\sqrt{2}$; $\sqrt{5}$.

CHAPTER XIX

THE PYTHAGOREAN THEOREM

***282. The square of the hypotenuse of a right triangle equals the sum of the squares of the other two sides.**

SUMMARY

283. State the formula for the hypotenuse c in terms of a and b.

State the formula for a in terms of c, if $\angle A = 60°$.

State the formula for c in terms of b, if $\angle A = 45°$.

State the four steps in the analysis of numerical exercises.

284. NUMERICAL EXERCISES

1. How long a ladder is required to reach a window 24 ft. high, if the lower end of the ladder is 10 ft. from the side of the house?

2. Find the altitude of an equilateral triangle whose side is 1 in.

3. Find the sides of a right triangle whose hypotenuse is 1 in., one of the acute angles being 60°.

4. Find the hypotenuse of a right triangle whose side is 1 in., one of the acute angles being 45°.

5. Find the length of the shortest chord and of the longest chord that can be drawn through a point 6 in. from the center of a circle whose radius is 10 in.

6. The distance from the center of a circle to a chord 10 ft. long is 12 ft. Find the distance from the center to a chord 24 ft. long.

7. The radius of a circle is 6 in. Find the length of the tangents drawn from a point 10 in. from the center, and also the length of the chord between the points of tangency.

8. The radius of a circle is 2 in. From a point 4 in. from the center, a secant is drawn so that its internal part is 1 in. Find the length of the secant.

9. If one angle of a right triangle is 60°, and the opposite side is $2\sqrt{3}$, find the other sides. (See Art. 120.)

10. If one angle of a right triangle is 45°, and the hypotenuse is 5 in., find the other sides.

11. A window 5 ft. wide has a circular arch at the top with a rise of 20 in. Find the radius of the circle of the arch.

12. The non-parallel sides of a trapezoid are each 53 units in length, and one of the bases is 56 units longer than the other. Find the altitude of the trapezoid.

NUMERICAL EXERCISES ON PROPORTION

1. The base and altitude of a triangle are 7 ft. 6 in. and 5 ft. 6 in. respectively. If the corresponding base of a similar triangle is 5 ft. 6 in., find the altitude.

2. The sides of a triangle are 6, 7, and 8. In a similar triangle, the side corresponding to 8 is 40. Find the other two sides.

3. The sides of a right triangle are 9, 12, and 15. Find the parts of the hypotenuse made by the altitude. Find this altitude.

4. The radius of a circle is 13 in. Through a point 5 in. from the center, a chord is drawn. What is the product of the two parts of the chord?

5. From the end of a tangent 20 in. long, a secant is drawn through the center of the circle. If the external part of the secant is 8 in., find the radius of the circle.

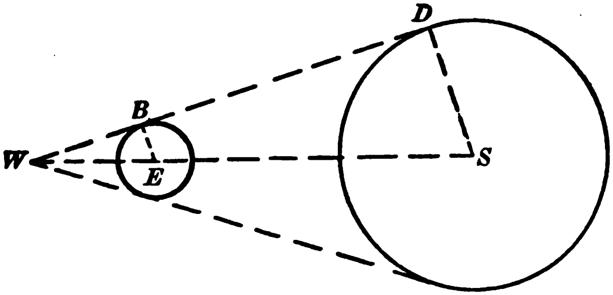
6. A tree casts a shadow 90 ft. long, when a post 6 ft. high casts a shadow 4 ft. long. How high is the tree?

7. Construct ex. 13, Art. 86, with similar triangles instead of congruent triangles.

8. Construct ex. 14, Art. 86, similarly.

9. Construct ex. 15, Art. 86, similarly.

10. When the sun shines upon the earth, the earth casts a shadow in space. This shadow is represented on paper by a triangle. Thus:



In the figure, S is the center of the sun; E is the earth; EW is the length of the shadow. Find EW. Given: ES = 92,000,000 miles; SD = 433,000 miles; EB = 4,000 miles.

CHAPTER XX

SIMILAR POLYGONS AND CIRCLES

The basic method is:

Similar polygons have their corresponding angles equal and their corresponding sides in proportion. (Art. 253.)

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*285. *Two polygons are similar, if they are composed of the same number of triangles, similar each to each, and similarly placed.*

*286. *Two similar polygons may be divided into the same number of triangles, similar each to each, and similarly placed.*

*287. Problem: *On a given line corresponding to a side of a given polygon, construct a polygon similar to the given polygon.*

288. *Two regular polygons of the same number of sides are similar.*

289. In a series of equal ratios, the sum of the antecedents is to the sum of the consequents as . . . what?

*290. *The perimeters of two similar polygons are to each other as any two corresponding sides.*

*291. *The perimeters of two regular polygons of the same number of sides are to each other as their radii and as their apothems.*

292. What is a **Constant**, a **Variable**, a **Limit?**

What is the usual variable considered in geometry?

293. *If two variables are always equal, and if each approaches a limit, their limits are equal.*

*294. **The circumferences of two circles are to each other as their radii.**

295. *The ratio of a circumference to its diameter equals the ratio of any other circumference to its diameter.*

296. What does π represent? What is the formula for the circumference (C) in terms of π?

297. Problem: *Show that* $\pi > 3$, *and* < 4.

Its commonly accepted value is what?

SUMMARY

298. State two methods of proving polygons similar.

299. NUMERICAL EXERCISES

1. The perimeters of two similar polygons are 200 ft. and 300 ft. If a side of the first is 24 ft., find the corresponding side of the second.

2. Find the circumference of a circle whose diameter is 8 ft.

3. The diameter of a bicycle wheel is 28 in. How many revolutions does the wheel make in going one mile?

4. Find the diameter of a carriage wheel that makes 264 revolutions in going half a mile.

5. Find the central angle of an arc 5 ft. 10 in. long, if the radius is 9 ft. 4 in.

6. Find the arc whose central angle is 40°, if the radius is 6 in.

7. Find the width of a circular ring between two concentric circles whose circumferences are 130 ft. and 85 ft. respectively.

8. A water-tank is cylindrical in form and 20 ft. in diameter. How long must a piece of strap iron be cut to make a band around it, allowing 20 in. for overlapping?

9. The earth is nearly 8,000 miles in diameter. What is the approximate length of the equator?

10. Two wheels are each 24 in. in diameter and their centers are 8 ft. apart. What is the length of the belt that runs around the two wheels?

11. A driving wheel 4 ft. in diameter makes 100 revolutions per minute. It is belted to a second wheel that makes 50 revolutions per minute. What is the diameter of the second wheel?

12. The side of a square is 4 ft. Find the circumference of its inscribed and circumscribed circles.

REVIEW QUESTIONS

1. Define Ratio; Proportion; Extremes and Means; Antecedents and Consequents; Alternation; Inversion; Composition, and Division in proportion.

2. What is meant by: The product of the means equals the product of the extremes? What is its converse?

3. A proportion may be considered as a fractional what?

4. If three terms of one proportion are equal respectively to three terms of another proportion, what can be said of the remaining terms?

5. What is the fundamental principle for all proportion of lines?

6. What is a more common form of this principle?

7. Define Internal division of a line; External division of a line?

8. Define Similar polygons.

9. State four methods of proving triangles similar.

10. Which is shorter, proving triangles similar or other polygons?

11. How can the corresponding sides of similar triangles be found?

12. State four methods of proving four lines in proportion.

13. There are now nine methods of proving lines parallel. Name most of them.

14. There are now seventeen methods of proving angles equal. Name as many of them as possible.

15. What is the usual method of getting four lines in proportion?

16. The ratio of two lines means the ratio of what?

17. State the three steps in proving that the product of two lines equals the product of two other lines.

18. Group 271, 272, and 273 under one statement.

19. Define Mean proportional; Third proportional; Fourth proportional.

20. If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, state three conclusions.

21. State three methods of finding a mean proportional to two given straight lines.

22. Represent algebraically the fourth proportional to a, b, and c.

23. Represent algebraically the mean proportional between a and b.

24. What is the Pythagorean theorem?

25. What is its formula?

26. State two important methods of proving polygons similar.

27. What conclusion can be made from a series of equal ratios?

28. What is a Constant? Variable? Limit?

29. What is the usual variable considered in geometry?

MISCELLANEOUS EXERCISES

1. If two transversals intersect between two parallel lines, the triangles formed are similar.

2. The diagonals of a trapezoid form with the bases two similar triangles.

3. If two circles are tangent internally and three lines are drawn through the point of tangency, the chords joining the ends of these lines form similar triangles.

4. If two circles are tangent externally and two lines are drawn through the point of tangency, the chords joining the ends of these lines form two similar triangles.

5. Prove the same theorem if the circles are tangent internally.

6. If triangle ABC is inscribed in a circle and AP is drawn to the middle point of arc BC, and cutting chord BC in D; triangle ABD is similar to triangle APC.

7. If triangle ABC is inscribed in a circle, and AD is drawn $\perp$ to BC, and the diameter AE is drawn; triangle ABD is similar to triangle AEC.

8. The median to the base of a triangle bisects any line cutting the triangle and parallel to the base.

9. The squares of the arms of a right triangle are to each other as their projections on the hypotenuse.

10. Two rhombuses are similar if an angle of the one equals an angle of the other.

11. Two parallelograms are similar if an angle of the one equals an angle of the other and the including sides are proportional.

12. Two rectangles are similar, if two adjacent sides of the one are in proportion with two corresponding sides of the other.

13. An interior common tangent of two circles divides the line of centers proportionally with the radii.

14. An exterior common tangent of two circles divides the line of centers externally into parts proportional with their radii.

15. If three or more straight lines drawn from a common point intersect two parallels, the corresponding parts of the parallels are proportional.

16. If two circles are tangent externally, their common external tangent is a mean proportional between their diameters.

17. If two circles are tangent internally, all chords of the greater circle drawn from the point of tangency are divided proportionally by the circumference of the smaller circle.

18. If in a right triangle ABC the altitude AD is drawn to the hypotenuse, $AD : AB :: AC : BC$.

19. If from any point E in the chord AB, EC is drawn perpendicular to the diameter AD, $AC \times AD = AB \times AE$.

20. If AB is a diameter, BD the tangent at B, and DA meets the circumference at E, $AB^2 = AE \times AD$.

21. Two triangles are similar if two sides and the median to one of these sides of the one are in proportion with the corresponding parts of the other.

22. Construct a triangle similar to a given triangle, having given its perimeter.

23. If one chord bisects another chord, either part of the second chord is a mean proportional between the parts of the first chord.

24. Draw a line through a given point so that it will cut off on the sides of a given angle parts having a given ratio.

25. Draw a line from a given point to a given line so that it will have a given ratio to the perpendicular from that point.

26. Draw a line through a given point and between the sides of a given angle so that it will be cut in a given ratio by the point.

27. Construct two lines, having given their sum and their ratio.

28. Construct two lines, having given their difference and their ratio.

29. Construct a circumference equal to the sum of two given circumferences.

30. Construct a circumference equal to the difference of two given circumferences.

31. Inscribe a square in a semicircle.

32. Inscribe a square in a triangle.

33. State and prove the converse of Art. 249.

34. The arms of a right triangle are 24 and 32; find the hypotenuse.

35. The side of a square is 12 ft. What is the diagonal?

36. The base of an isosceles triangle is 16 and the altitude is 15. Find the sides.

37. The tangent to a circle from an outside point is 12 in. and the radius is 5 in. What is the length of the line from the outside point to the center?

38. The distance of a chord 14 in. long from the center is 12 in. Find the radius.

39. The radius of a circle is 13 ft. What is the length of a chord 5 ft. from the center?

40. The arms of a right triangle are 8 and 15. Compute the hypotenuse and the altitude upon the hypotenuse.

41. The hypotenuse of a right triangle is 4 ft. 4 in. and one arm is 1 ft. 8 in. Find the other arm.

42. The base of an isosceles triangle is 90 ft. and the equal sides are each 53 ft. Find the altitude.

43. The radius of a circle is 9 ft. 2 in. Find the length of the tangent from a point 12 ft. 2 in. from the center.

44. The distance of a chord 24 in. long from the center is 5 in. Find the distance of a chord 10 in. long from the center.

45. Each side of an equilateral triangle is 8 ft. Find the altitude.

46. The altitude of an equilateral triangle is 16 in. Find a side.

NOTE: Let $2x$ = side. Solve the right triangle.

47. If two sides of a triangle are 5 yds. and 8 yds. and the included angle is 60°, what is the third side? (Draw altitude to side 5. Art. 120.)

48. If two sides of a triangle are 7 and 8 and the included angle is 120°, what is the third side?

49. Find the altitude of an isosceles triangle whose base is 8 in. and whose side is 5 in.

50. Find the side of an isosceles right triangle whose base is 8.

51. If the length of the common chord of two intersecting circles is 16 and their radii are 10 and 17, what is the distance between their centers?

52. The line of centers of two circles is 10 and their radii are 4 and 13. Find the length of the common chord.

53. The diagonal of a rectangle is 41 and one side is 40. Find the other side.

54. The diagonal of a square is 20 in. Find the side.

55. The radius of a circle is 10, and the length of a tangent is 26. What is the distance from the end of the tangent to the center.

56. The diameters of two concentric circles are 22 in. and 122 in. Find the length of a chord of the larger which is tangent to the smaller.

57. The lower ends of a post and a flag pole are 42 ft. apart; the post is 8 ft. high and the pole is 48 ft. What is the length of a rope connecting their tops?

58. The radii of two circles are 16 and 34 in., and the line of centers is 82. Find the length of their common external tangent; and of their common internal tangent.

59. From a point 24 ft. above sea-level, one can see about 6 miles. Find the diameter of the earth.

60. A ladder 32 ft. 6 in. long is placed so that it just reaches a window 26 ft. above the street; and when turned about its foot, just reaches a window 16 ft. 6 in. from the street on the other side. Find the width of the street.

61. Find the length of the diagonal of a square whose side is 5 units.

62. One of the non-parallel sides of a trapezoid is perpendicular to the bases. If the length of this side is 40, and of the bases 31 and 22, what is the length of the other side?

63. The non-parallel sides of a trapezoid are each 25 in. in length, and one base is 40 in. longer than the other base. Find the altitude.

64. Two secants are drawn to a circle from an outside point. If their external parts are 24 and 18, while the internal part of the first is 16, what is the internal part of the second?

65. Two parallel chords on opposite sides of the center of a circle are 48 and 14, and distance between their middle points is 31. What is the radius?

66. Also find the length of a chord parallel to the given chords and midway between them. Find the distance between the chords if they are on the same side of the center.

67. If in triangle ABC, $\angle A$ is a right angle, and AD is the altitude upon the hypotenuse BC, $AB = 15$, $AD = 9$; find BD, BC, and AC.

68. $AD = 8$, $DC = 4$; find AC, BC, and AB.

69. $AB = 2$, $DC = 3$; find BC, BD, and AD.

70. $AB = 9$, $AC = 12$; find BC, BD, and AD.

71. $BD = 6$, $DC = 24$; find AB, AD, and AC.

72. The line joining the mid-point of a chord to the mid-point of its arc is 5 in. If the chord is 24 in. long, what is the diameter?

73. If the chord of an arc is 30 and the chord of its half is 17, what is the diameter?

74. To a circle whose radius is 10, two tangents are drawn from a point, each 24 units long. Find the length of the chord joining the points of contact.

75. The parts of one chord made by a second chord are 4 and 27. The first part of the second chord is 6, what is the other part?

76. One of two intersecting chords is 38 in. long, and the parts of the other are 10 in. and 24 in. Find the parts of the first chord.

77. Find the product of the parts of any chord drawn through a point 9 units from the center of a circle whose diameter is 24 units.

78. A tangent to a circle is 12, and the secant from the same point is 18. Find the internal part of the secant.

79. The internal part of a secant 50 in. long is 32 in. Find the tangent from the same point.

AREAS

CHAPTER XXI

AREAS OF SIMPLE FIGURES

The basic method is:

300. **The area of a rectangle equals the product of the base and the altitude.**

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301. *The area of a square equals the square of its side.*

*302. **The area of a parallelogram equals the product of the base and the altitude.**

*303. **The area of a triangle equals one-half the product of the base and the altitude.**

304. *The area of a trapezoid equals one-half the sum of the bases multiplied by the altitude.*

305. State a method of finding the area of a polygon.

*306. *The area of a regular polygon equals one-half the product of the perimeter and the apothem.*

*307. **The area of a circle equals one-half the product of the circumference and the radius.**

308. Derive the formula for the area of a circle in terms of π.

309. *The area of a sector equals one-half the product of its arc and the radius.*

FORMULAS

310. Let h = altitude; b = base; b^1 = upper base; m = mid-line of trapezoid; P = perimeter; r = apothem; C = circumference; R = radius. State the formula for the area of:

a. A parallelogram.
b. A triangle.
c. A trapezoid.
d. A regular polygon.
e. A circle.
f. An equilateral triangle.

311. EXERCISES

1. The area of a rhombus equals one-half the product of its diagonals.

2. The area of a triangle equals one-half the product of the perimeter and the radius of the inscribed circle.

3. The area of any circumscribed polygon equals one-half the product of its perimeter and the radius of the circle.

4. If the perimeter of a rectangle is 60 units, and the length is twice the width, find the area.

5. A canal is 14 ft. deep, 60 ft. wide at the top, and 45 ft. wide at the bottom. Find the area of a cross-section.

6. A rhombus contains 100 sq. ft., and the length of one diagonal is 20 ft. Find the other diagonal.

7. Find the area of a right triangle, if the hypotenuse is 34, and one side is 16.

8. Find the area of an equilateral triangle, if one side is 8.

9. A house is 80 ft. long, 60 ft. wide, 50 ft. high to the eaves, 70 ft. high to the ridgepole. Find the entire surface area.

10. Find the area of a circle with radius 4 ft.

11. The area of a circle is 196 sq. in. Find the radius.

12. Find the area of a sector whose central angle is 40°, and radius 6 in.

13. The area of an equilateral triangle with side a equals $\frac{a^2}{4}\sqrt{3}$.

14. The area of a right triangle with one angle 60° equals $\frac{a^2}{8}\sqrt{3}$ where a is the hypotenuse.

15. The area of an isosceles right triangle equals $\frac{a^2}{4}$ where a is the hypotenuse.

CHAPTER XXII

EQUALITY OF AREAS

The two basic methods are:

1. **Prove their area formulas equal.**
2. **Prove the figures congruent.**

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312. **Two parallelograms are equal if they have equal bases and equal altitudes.**

313. *Two triangles are equal if they have equal bases and equal altitudes.*

314. *Two trapezoids are equal if they have equal mid-lines and equal altitudes.*

315. *A triangle equals one-half a parallelogram if they have equal bases and equal altitudes.*

*316. *The square on the hypotenuse of a right triangle equals the sum of the squares on the other two sides.*

317. *The square on the hypotenuse of a right triangle minus the square on a second side equals the square on the third side.*

*318. PROBLEM: *Construct a square equal to the sum of two given squares.*

319. PROBLEM: *Construct a square equal to the difference of two given squares.*

*320. PROBLEM: *Construct a triangle equal to a given polygon.*

321. How can a^2 be represented in Geometry? How can ab? How can $\frac{1}{2}ab$?

*322. PROBLEM: *Construct a square equal to a given parallelogram.*

SUMMARY

323. State three methods of proving areas equal.

324. EXERCISES

1. The non-parallel sides of a trapezoid form with the parts of the diagonals two equal triangles.

2. The median of a triangle divides the triangle into two equal triangles.

3. The line joining the middle points of two sides of a triangle cuts off a triangle equal to one-fourth of the triangle.

4. The line joining the middle points of two adjacent sides of a parallelogram cuts off a triangle equal to one-eighth of the parallelogram.

5. Divide a triangle into three equal triangles by drawing lines from any one vertex.

6. Construct a square equal to a given triangle.

7. Construct an isosceles triangle equal to a given triangle and having the same base.

8. What is the locus of the vertices of all equal triangles on the same base?

9. The bases of a trapezoid are 16 ft. and 20 ft., and the altitude is 12 ft. Find the base of an equal rectangle having the same altitude.

10. The base of a triangle is 15 ft. and the altitude 8 ft. Find the perimeter of an equal rhombus with altitude 6 ft.

11. Upon the diagonal of a rectangle 12 ft. by 5 ft. a triangle is constructed equal to the rectangle. What is its altitude?

12. Find the side of a square equal to a trapezoid with bases 28 ft. and 22 ft., and each non-parallel side 5 ft.

13. A trapezoid equals a triangle having the same altitude and a base equal to the sum of its bases.

14. A square equals one-half the square on its diagonal.

15. The diagonals of a parallelogram make four equal triangles.

16. An isosceles right triangle equals one-fourth the square on its hypotenuse.

17. A right triangle with one angle 60° equals one-half the equilateral triangle on its hypotenuse.

18. Any line through the intersection of the diagonals of a parallelogram divides the parallelogram into two equal figures.

19. If two triangles have equal bases and equal altitudes, lines parallel to the bases and equally distant from the vertices cut off equal triangles.

CHAPTER XXIII

PROPORTION IN AREAS

The basic method is:
Divide their area formulas.

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325. **Two parallelograms are to each other as the products of their bases and altitudes.**

326. *Two parallelograms with equal bases are to each other as their altitudes.*

327. *Two parallelograms with equal altitudes are to each other as their bases.*

328. *Two triangles are to each other as the products of their bases and altitudes.*

329. *Two triangles with equal bases are to each other as their altitudes.*

330. *Two triangles with equal altitudes are to each other as their bases.*

*331. *Two triangles, having an angle of the one equal to an angle of the other, are to each other as the products of the sides including the equal angles.*

*332. *Two similar triangles are to each other as the squares of any two corresponding sides.*

*333. *Two similar polygons are to each other as the squares of any two corresponding sides.*

*334. *Two similar regular polygons are to each other as the squares of their radii, or as the squares of their apothems.*

*335. *The areas of two circles are to each other as the squares of their radii.*

SUMMARY

336. State three methods of proving areas in proportion.

337. EXERCISES

1. Draw a line from a vertex of a triangle to a point in the opposite side which shall divide the triangle into two triangles in the ratio of 3 to 5.

2. The areas of two similar triangles are to each other as the squares of the corresponding altitudes; medians; angle-bisectors.

3. The base of a triangle is 16 ft. and the altitude is 25 ft. What is the area of a triangle cut off by a line parallel to the base and 15 ft. from the base?

4. In two similar polygons, two corresponding sides are 3 and 5. The area of the first polygon is 90. Find the area of the second polygon.

5. If one of two similar polygons is double the other, what is the ratio of their corresponding sides?

6. From a given rectangle, cut off a rectangle whose area is two-thirds that of the given rectangle.

7. Two rectangles have equal altitudes, and bases 15 and 3 respectively. What is the ratio of their areas?

8. The equilateral triangle drawn on the hypotenuse of a right triangle equals the sum of the equilateral triangles drawn on the other two sides.

9. Find the side of an equilateral triangle whose area is four times the area of an equilateral triangle whose side is 3 in.

10. The diameters of two circles are 4 ft. and 9 ft. respectively. What is the ratio of their areas?

11. The diameter of a circle is 15 in. Find the diameter of a circle twice as large.

12. If the area of one circle is four times that of another, and the radius of the first is 6 in., what is the radius of the second?

13. The area of a circle is four times the area of the circle drawn on its radius as a diameter.

14. The area of an inscribed equilateral triangle is one-fourth the area of the circumscribed equilateral triangle.

15. The area of an inscribed square is one-half the area of the circumscribed square.

16. The area of an inscribed regular hexagon is three-fourths the area of the circumscribed regular hexagon.

17. The area of an inscribed regular hexagon is equal to twice the area of an inscribed equilateral triangle.

REVIEW QUESTIONS

1. What is the unit of measure for areas?

2. State the basic theorem for areas.

3. The proof of the area of a circle depends upon what two theorems?

4. State the formula for the area of a parallelogram.

5. The area of a triangle equals what?

6. The area of a trapezoid equals what? Also in terms of the mid-line and altitude?

7. The area of a regular polygon equals what?

8. The area of a circle equals what in terms of the circumference and radius? Also in terms of π.

9. Art. 316 is what theorem proved once before?

10. The former proof was by algebra. This proof is by what?

11. What geometrical figure does a^2 represent? ab? $\frac{1}{2}ab$?

12. State a method of proving triangles equal but not necessarily congruent.

13. The perimeters of two similar polygons are to each other as what?

14. The areas of two similar polygons are to each other as what?

15. Define Similar Sectors.

16. The ratio of a sector to its circle is determined by what?

17. The circumferences of two circles are to each other as what?

18. The areas of two circles are to each other as what?

19. In what two kinds of right triangle can the sides be found if only one side is known?

20. Represent algebraically the altitude of an equilateral triangle whose side is a; its area.

21. Represent algebraically the diagonal of a square whose side is a.

22. How can a square be constructed equal to a given triangle?

23. How can a square be constructed equal to a given polygon?

24. What is a method of finding the area of an irregular polygon?

25. In proving areas equal, what methods can be used?

26. If the area of a circle equals πR^2, what does R equal?

27. If the radius of one circle is twice the radius of another circle, what can be said of their circumferences? Their areas?

28. If the number of sides of a regular circumscribed polygon is indefinitely increased, what lines become variables, and approach what lines as limits?

29. How many times has the theorem of limits been used, and in what theorems?

30. In proving areas in proportion, what methods can be used?

CHAPTER XXIV

(Supplementary)

MENSURATION OF THE TRIANGLE

The basic method is:

The square of the hypotenuse of a right triangle equals the sum of the squares of the other two sides. (Art. 282.)

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338. What is the **Projection** of a point upon a line? A line upon a line?

339. *The square of the side opposite an acute angle of a triangle equals the sum of the squares of the other two sides minus twice the product of one of those sides and the projection of the other upon it.*

340. *The square of the side opposite an obtuse angle of a triangle equals the sum of the squares of the other two sides plus twice the product of one of those sides and the projection of the other upon it.*

341. Group articles 282, 339, and 340 under one general statement.

342. *The sum of the squares of two sides of a triangle equals twice the square of half the third side plus twice the square of the median to that side.*

343. *The product of two sides of a triangle equals the diameter of the circumscribed circle multiplied by the altitude to the third side.*

344. *The product of two sides of a triangle equals the square of the bisector of the included angle plus the product of the parts of the third side made by the bisector.*

345. *If* s *is the semi-perimeter of a triangle with sides* a, b, *and* c, *its area* $= \sqrt{s(s-a)(s-b)(s-c)}$.

346. *If AB is a side of a regular inscribed polygon with radius R, the side of a regular inscribed polygon of double the number of sides* $= \sqrt{2R^2 - R\sqrt{4R^2 - \overline{AB}^2}}$.

347. PROBLEM: *Compute the value of* π.

SUMMARY

348. If the three sides of a triangle are known, state the formula for finding:

a. Whether the angles are acute, right, or obtuse.
b. The projection of one side upon another.
c. An altitude.
d. The medians.
e. The diameter of the circumscribed circle.
f. The angle-bisectors.
g. The area.

349. EXERCISES

1. If the radius of a circle is 1 in., show that: In a regular inscribed $\triangle$, (a side) $a = \sqrt{3}$; (apothem) $r = \frac{1}{2}$; $\angle A = 60°$.

2. Also, in a regular inscribed square, $a = \sqrt{2}$; $r = \frac{1}{2}\sqrt{2}$; $\angle A = 90°$; $\angle BOC = 90°$.

3. Also, in a regular inscribed hexagon, $a = 1$; $r = \frac{1}{2}\sqrt{3}$; $\angle A = 120°$; $\angle BOC = 60°$.

4. Also, in a regular circumscribed $\triangle$, $a = 2\sqrt{3}$; (radius of $\triangle$) $R = 2$.

5. Also, in a regular circumscribed square, $a = 2$; $R = \sqrt{2}$.

6. Also, in a regular circumscribed hexagon, $a = \frac{2}{3}\sqrt{3}$; $R = \frac{2}{3}\sqrt{3}$.

7. If the sides of a triangle are 3, 4, and 5, is the angle opposite 5 acute, right, or obtuse?

8. If the sides of a triangle are 7, 8, and 12, is the angle opposite 12 acute, right, or obtuse? Opposite 7?

9. If the sides of a triangle are 6, 9, and 12, find the altitudes; the medians; the angle-bisectors; the diameter of the circumscribed circle.

10. If the sides of a triangle are 10, 14, and 16, find the median and the altitude to 14.

11. If the sides of a triangle are 4, 5, and 6, find the bisector of the angle opposite 6 and the altitude to 6.

12. Find the area of the triangle in ex. 7; ex. 8; ex. 9; ex. 10; ex. 11.

13. Find the diameter of the circumscribed circle in ex. 7, ex. 8, ex. 9, ex. 10, ex. 11.

14. If AB is a side of a regular inscribed polygon with radius 1, derive the formula for the side of a regular inscribed polygon of double the number of sides.

15. If the side of a regular inscribed hexagon is 1, find the side of a regular inscribed dodecagon.

16. The sum of the squares of the sides of a parallelogram equals the sum of the squares of the diagonals.

17. If the sum of the squares of two sides of a triangle equals the square of the third side, the triangle is a right triangle.

REVIEW QUESTIONS

1. State the theorem of limits.

2. In what kind of figure is it used?

3. What does π represent? What is its commonly accepted value?

4. State the formula for the circumference in terms of π.

5. How can a line of length $\sqrt{3}$ be constructed?

6. In similar triangles, are any two corresponding lines in proportion with any other two corresponding lines?

7. How is the area of a sector found?

8. How is the area of a segment of a circle found?

9. State the formula for the diagonal of a square in terms of its side.

10. State the formula for the altitude of an equilateral triangle.

11. State the formula for the area of an equilateral triangle.

CHAPTER XXV

(Supplementary)

FURTHER RATIOS

350. Define **Extreme and Mean Ratio.**

351. PROBLEM: *Divide a line in extreme and mean ratio.*

352. PROBLEM: *Construct an angle of* 36°.

353. PROBLEM: *Inscribe in a circle a regular decagon* (10 sides).

354. PROBLEM: *Construct a square having a given ratio to a given square.*

355. PROBLEM: *Construct a polygon similar to a given polygon and having a given ratio to it.*

356. PROBLEM: *Construct a polygon similar to a given polygon and equal to a second given polygon.*

357. What are incommensurable quantities? (Art. 202.) State the theorem of limits. (Art. 293.)

358. *In the same or equal circles two central angles have the same ratio as their intercepted arcs.*

359. *Three or more parallel lines intercept proportional parts on two transversals.* (The incommensurable case of Art. 237.)

360. *The area of a rectangle equals the product of its base and altitude.* (The incommensurable case of Art. 300.)

361. EXERCISES

1. Divide internally in extreme and mean ratio a line 20 meters long; externally.

2. Divide internally in extreme and mean ratio a line 4 ft. long; externally.

3. Inscribe a regular pentagon (5 sides) in a circle.

4. Inscribe a regular pentadecagon (15 sides) in a circle. (Construct central angles of 24°. That is, 60° – 36°.)

5. Construct a square $\frac{3}{4}$ as large as a given square.

6. Construct a square $\frac{2}{3}$ as large as a given square.

7. Construct a circle $\frac{4}{5}$ as large as a given circle.

8. Construct a circle five times as large as a given circle.

MISCELLANEOUS EXERCISES

1. Find the area of a triangle whose sides are 13, 14, and 15.

2. What is the area of a parallelogram whose base is 9 in. and altitude 4 in.? What is the area of a triangle having the same base and altitude?

3. The base of a triangle is 12 and its altitude is 9. The bases of an equal trapezoid are 13 and 14; what is its altitude?

4. The base of a triangle is 8. Find the base of a similar triangle 5 times as large; twice as large.

5. If similar polygons are constructed upon the three sides of a right triangle, the polygon on the hypotenuse equals the sum of the other two.

6. If through the middle point of one of the non-parallel sides of a trapezoid a line is drawn parallel to the opposite side, the parallelogram formed is equal to the trapezoid.

7. If two equal triangles have the same base and lie on opposite sides of it, the line ajoining their vertices is bisected by the base.

8. Two triangles having an angle of the one supplementary to an angle of the other are to each other as the products of the sides including the supplementary angles.

9. Show by drawing a geometric figure that:
$(a + b)^2 = a^2 + 2\,ab + b^2.$

10. $(a - b)^2 = a^2 - 2ab + b^2.$

11. $(a + b)\ (a - b) = a^2 - b^2.$

12. $a\ (a + b) = a^2 + ab.$

13. $(\frac{a}{2})^2 = \frac{a^2}{4}.$

14. Find the area of a triangle whose sides are 7, 10, and 13.

15. Find the area of a triangle whose sides are 17, 25, and 28.

16. Find the three altitudes of a triangle whose sides are 7, 10, and 11.

17. Find the three altitudes of a triangle whose side are 8, 12, and 16.

18. Find the area of an equilateral triangle whose side is 12.

19. Find the area of an equilateral triangle whose altitude is 6.

20. Find the side of an equilateral triangle whose area is $81\sqrt{3}$.

21. Find the radius of the inscribed circle and the radius of the circumscribed circle of the triangle 7, 10, 11.

22. Find the radius of the inscribed circle and the radius of the circumscribed circle of the triangle 9, 10, 11.

23. The base of a parallelogram is 1 ft. 6 in. and its altitude is 2 in. Find the area. Find the side of an equal square.

24. The area of a rectangle is 540 and its base is 36. Find its altitude and diagonal.

25. The base of a rectangle is 3 ft. 4 in. and its diagonal is 3 ft. 5 in. Find its area.

26. Find the altitude of an equilateral triangle whose area is 360 square units.

27. The area of a polygon is 216 and its shortest side is 4. Find the area of a similar polygon whose shortest side is 5. Find the shortest side of a similar polygon three times as large.

28. The bases of a trapezoid are 2 ft. 2 in. and 4 ft. 4 in. and the altitude is 3 ft. 3 in. Find the area.

29. The area of a trapezoid is 720 sq. ft. and its bases are 3 ft. and 4 ft. 6 in. Find the altitude.

30. The equal sides of an isosceles triangle are each 17 in. and the base is 16 in. Find the area.

31. Find the area of an isosceles right triangle whose hypotenuse is 60.

32. Find the area of a square whose diagonal is 15.

33. The diagonals of a rhombus are 6 in. and 8 in. Find the area; perimeter; and altitude.

34. The sides of two equilateral triangles are 33 and 56 respectively. Find the side of an equilateral triangle equal to their sum.

35. The side of an equilateral triangle is 8. Find the side of an equilateral triangle equal to four times the first triangle.

36. Two similar polygons have corresponding sides equal to 7 and 24. Find the corresponding side of a third similar polygon equal to their sum.

37. The bases of a trapezoid are 28 and 22 and the non-parallel sides are each 5. Find the area.

38. Find the side of a square whose area is 961.

39. The projections of the arms of a right triangle upon the hypotenuse are 4 and 9. Find the area.

40. The base of a triangle is 20 in. and its altitude is 8 in. Find the area of a triangle cut off by a line parallel to the base and 4 in. from it; also 2 in. from it.

41. Find the area of a polygon circumscribed about a circle of radius 3. The perimeter of the polygon is 20.

42. The bases of a trapezoid are 6 and 11, and its altitude is 2. If the non-parallel sides are produced till they meet, find the area of the smaller triangle.

43. Find the area of a square inscribed in a circle whose radius is 9 ft.

44. The area of an isosceles right triangle is 162 sq. in. Find its hypotenuse. (Let x equal each arm.)

45. The perimeter of a polygon is 60 and the radius of the inscribed circle 5. Find the area of the polygon.

46. The sides of a triangle are 15, 14, 13. Find the area, the altitudes, the radii of the inscribed and circumscribed circles.

47. Also for 25, 63, 74.

48. One diagonal of a rhombus is five-thirds of the other, and the difference of the diagonals is 8. Find the area.

49. A trapezoid is composed of a rhombus and an equilateral triangle. Each side of each figure is 8 ft. Find the area of the trapezoid.

50. In a triangle whose base is 40 and altitude is 24, a line is drawn parallel to the base and bisecting the triangle. Find the distance from the base to this parallel.

51. The area of a polygon is 324 and one side is 18. Find the corresponding side of a similar polygon whose area is 196.

52. The sides of a triangle are 6, 7, and 8. Find the areas of the two parts into which the triangle is divided by the bisector of the angle between 6 and 7.

53. Construct a triangle equal to a given triangle and having the same base and an angle adjoining the base.

54. Construct a parallelogram equal to a given parallelogram, having the same base and an angle adjoining the base.

55. Construct a right triangle equal to a given triangle.

56. Construct a right triangle equal to a given triangle, having the hypotenuse equal to one side of the given triangle.

57. Construct a right triangle equal to a given square.

58. Construct a square equal to a given right triangle.

59. Construct a square equal to the sum of two given right triangles.

60. Construct a triangle equal to a given triangle, having its vertex angle equal to a given angle.

61. Construct a rhombus equal to a given parallelogram.

62. Construct a parallelogram equal to a given triangle.

63. Construct a square equal to a given trapezoid.

64. Construct an isosceles right triangle equal to a given triangle.

65. Construct a square equal to a given rhombus.

66. Construct a square equal to the difference of two given parallelograms.

67. Construct a square equal to the sum of several given triangles.

68. Construct a square equal to the sum of several given polygons.

69. Construct a rectangle having a given altitude and equal to a given rectangle.

70. Construct a triangle having a given base and equal to a given triangle.

71. Construct a rectangle having a given base and equal to a given triangle.

72. Construct a triangle having a given base and equal to a given polygon.

73. Construct a square having twice the area of a given square.

74. Construct a triangle three times as large as a given triangle.

75. Construct a triangle three times as large as a given similar triangle. Four times. Five times.

76. Construct an isosceles triangle equal to a given square and having a given base.

77. Draw a line parallel to the base of a triangle forming a triangle equal to one-third the given triangle. One-fourth. One-fifth.

78. Divide a triangle into two equal parts by a line drawn from any point in a side.

79. Divide a triangle into three equal parts by lines drawn from any point in a side.

80. Bisect a triangle by a line perpendicular to a side.

81. Bisect a parallelogram by a line perpendicular to a side.

82. Find the area of a square whose perimeter is 80 ft.

83. Find the area of a regular hexagon whose side is 4 and apothem is $2\sqrt{3}$.

84. Find the area of a regular dodecagon (12 sides) whose side is $10\sqrt{2-2\sqrt{3}}$ and whose apothem is $5\sqrt{2+\sqrt{3}}$.

85. The circumference of a circle equals 12π. Find the radius and area.

86. The radius of a circle is 20. Find the circumference and area.

87. The diameter of a circle is 50 ft. Find the circumference and area.

88. Find the radius of a circle whose area is 100 sq. miles.

89. Construct a regular hexagon equal to one-half a given regular hexagon.

90. Construct a regular octagon upon a given line as side. Decagon. Dodecagon.

91. Construct a circle equal to five times a given circle.

92. Construct a circle equal to the sum of two given circles; also equal to their difference.

93. Construct a circle equal to one-half a given circle.

94. Within a given circle construct a concentric circle equal to one-half the given circle.

95. Construct a circle equal to the area bounded by two concentric circumferences.

96. The medians of a triangle divide it into six equal triangles.

97. The carrying capacity of four pipes each 3 in. in diameter is the same, neglecting friction, as that of one pipe of what diameter?

98. The diameter which bisects a chord, whose length is 168 units, is 175 units long. Find the distances from the ends of the chord to the ends of the diameter.

99. The shadow of a yardstick perpendicular to the ground is 4 ft. long. Find the height of a tree whose shadow is 100 yards.

100. The bases of a trapezoid are 6 and 10 and the altitude is 4; the non-parallel sides are produced until they meet. Find the altitude of the larger triangle.

101. The diagonals of a trapezoid, whose bases are AD and BC, intersect at E. If $AE = 18$, $EC = 6$, $BD = 32$, find BE and ED.

102. The sides of a triangle are 6, 9, 11. Find the parts of 6 made by the bisector of the opposite angle.

103. The sides of a triangle are 9, 12, and 16. Find the parts of 9 made by the bisector of the opposite exterior angle.

104. If the sides of a triangle are 6, 8, and 10, and the shortest side of a similar triangle is 15, find the other sides.

105. The corresponding altitudes of two similar triangles are 3 and 5 and the base of the first is 7. What is the base of the second?

106. The perimeters of two similar polygons are 36 and 60; the shortest side of the first is 2. Find the shortest side of the second.

107. If two sides of a triangle equal 15 and 25 and the projection of 15 upon 25 is 9, what is the third side?

108. The sides of a triangle are 6, 8, and 9. Find the projection of 6 upon 8, and of 8 upon 9.

109. Two sides of a triangle are 32 and 24 units and include an angle of 60°. Find the third side.

110. Two sides of a triangle are 40 and 60 units and include an angle of 45°. Find the third side.

111. The sides of a triangle are 5, 6, and 9. Find the projection of 6 upon 5; and of 9 upon 5.

112. The sides of a triangle are 4, 13, and 15. Find the three altitudes.

113. In the previous figure, find the diameter of the circumscribed circle.

114. The sides of a triangle are 9, 10, and 17. Find the three altitudes.

115. The sides of a triangle are 4, 7, and 9. Find the three medians.

116. The sides of a triangle are 7, 8, and 9. Find the median to 8.

117. In triangle ABC, $AB = 8$, $AC = 11$, the median to BC equals $8\frac{1}{2}$. Find BC.

118. The sides of a triangle are 3, 6, and 7. Find angle-bisector to 7.

119. The sides of a triangle are 7, 15, and 20. Find the bisector of the smallest angle.

120. If two sides of a triangle are 9 and 12, and the diameter of the circumscribed circle is 15, find the third side.

121. Given a line one unit in length, construct a line equal to $\sqrt{2}$ units; to $\sqrt{5}$ units; to $\sqrt{3}$ units.

122. Construct a line equal to the $\sqrt{10}$ in; $2\sqrt{6}$ in.

123. Find the angle and the central angle of: a regular pentagon; regular hexagon; regular octagon; regular decagon; regular dodecagon.

124. Find the length of an arc subtended by a side of an inscribed equilateral triangle in a circle whose radius is 5; of a square; of a regular inscribed hexagon; of a regular inscribed decagon; of a regular inscribed pentadecagon.

125. If the circumference of a circle is 110 yds., what is its diameter?

126. The radius of a circle is 9. What is the radius of a second circle whose circumference is twice the first circumference?

127. If the length of a quadrant is 8 meters, what is the radius?

128. A locomotive wheel is 7 ft. in diameter. How many revolutions will it make in running a mile?

TYPES OF COLLEGE EXAMINATION QUESTIONS

1. The area of the inscribed equilateral triangle equals one-half the area of the inscribed regular hexagon.

2. The area of a square inscribed in a sector, whose central angle is a right angle, equals one-half the square on the radius.

3. In a circle whose radius is 20, are inscribed an equilateral triangle, a square, and a regular hexagon. Find the perimeter, apothem, and area of each.

4. About a circle whose radius is 10 are circumscribed an equilateral triangle, a square, and a regular hexagon. Find the perimeter and area of each.

5. The circumference of a circle equals 30. Find the circumference of a circle having twice the area of the given circle.

6. The radius of a circle is 18. What is the radius of a second circle whose circumference is twice as long? What is the radius of a third circle whose area is twice as great?

7. Two concentric circles have their circumferences equal to 15 ft. and 20 ft. Find the area bounded by the two circumferences.

8. Find the area of a sector whose radius equals 15 and whose central angle is 40°.

9. The angle of a sector is 72° and its arc is 22 ft. What is its area?

10. A square is inscribed in a circle of radius 5. Find the area of a segment cut off by a side of the square.

11. In a circle whose radius is 10, what is the area of a segment whose central angle is 120°? 60°?

12. From a point without a circle of radius 35 two tangents are drawn forming an angle of 60°. Find the area of the figure formed by the tangents and the nearer arc.

13. If the hypotenuse of a right triangle is the base, and three semicircles are drawn, one on each side as a diameter and above the base, the sum of the two crescents thus formed equals the area of the triangle.

14. Prove this in the right triangle 6, 8, 10.

15. If the radius of a circle is 6 in., find the side and area of the inscribed square; of the inscribed regular hexagon.

16. If the diameter of a circle is 14 ft., and the diameter is divided into parts 6 ft. and 8 ft., and on these parts as diameters semi-circumferences are drawn on opposite sides of the diameter, the compound curve thus formed divides the circle in the ratio of 3:4.

17. Find the area bounded by three arcs of 60° and radius 5, if the concave sides of the arcs are turned within.

18. Two circles C and C' are tangent to each other externally and each is tangent to a straight line L. The line of centers makes an angle of 30° with L. If the radius of the larger circle is 3 in., find the radius of the smaller circle.

19. In an isosceles triangle ABC the sides AC and BC are each 25 in. long, and AB is 30 in. long. How far is A from the side BC?

20. The area bounded by two concentric circles equals the area of a circle whose diameter is that chord of the larger circle tangent to the smaller.

21. The radius of a circle is 4 ft. What is the area of that part of the circle outside the inscribed regular hexagon?

22. A square $ABCD$ has side of 24 in. Using A, B, C, and D as centers and radii equal to 8 in., four quadrants are drawn within the square. Find the perimeter and area of the figure with the corners cut off.

23. In the same square using A, B, C, and D as centers and radii equal to 12, four arcs are drawn without the square. Find the perimeter and area of the figure bounded by the four arcs.

24. Do the last two exercises but with an equilateral triangle of side 24 in.

25. A square has a side of 4 ft. If semi-circumferences are drawn on each side as a diameter and within the square, find the areas of the four spaces, bounded each by two quadrants.

26. A square has a side of 12 ft. Semi-circumferences are drawn on each side as a diameter and without the square. Find the area of the figure.

27. Do the same with an equilateral triangle.

28. Three equal circles are drawn each tangent to the other two. If the common radius is 5, find the area of the space between the circles.

29. Three equal water pipes are so placed that each touches the other two, and a string is tied around them. If the length of the string is 10 ft., find the radius of the pipes.

30. In the figure of ex. 29, if the radius of the pipes is 4 in., find the length of the string around them.

31. A circular arch of masonry, radius 25 ft., rests on two stone piers which are 40 ft. apart. Find the height of the center of the arch above the level of the top of the piers.

32. The sum of the areas of two similar triangles is 255 sq. in., and the ratio of their sides is 1 : 4. Find the area of each.

33. A roadway 60 ft. wide is cut through the middle of a circular field 120 ft. in diameter. Find the area of the remainder of the field.

34. A circular grass plot 12 ft. in diameter is cut by a straight path 3 ft. wide, one edge of which passes through the center of the plot. What is the area of the remaining grass plot?

35. A quarter-mile race track has parallel sides and semi-circular ends of radius 105 ft. Find the length of the parallel sides.

36. If the diameter of the earth is 8,000 miles, how far can the light of a lighthouse 150 ft. high be seen?

37. The minute hand of a clock is 2 in. long. How far does the end of it move in 25 minutes?

38. A and B are two points on a railway curve which is an arc of a circle. If the length of the chord AB is 200 ft. and the shortest distance from the midpoint of the curve to the chord is 4 ft., find the radius of the arc.

39. A right triangle ABC has a fixed hypotenuse AB, 2 in. long. The vertex of the right angle is allowed to take all possible positions. Construct accurately the locus of the mid-point P of the leg AC.

40. Through a point A within a circle whose center is B, chords are drawn. Construct the locus of the mid-points of these chords.

41. A chord of length 8 in. moves with its extremities always on the circumference of a circle whose radius is 5 in. The chord is trisected by the points P and Q. What is the locus of P? Find the length of this locus.

42. A circle of radius 2 in. rolls around a square whose side is 4 in. Construct the locus of the center of the circle and find to two decimal places both the length of the locus and the area enclosed by it.

43. Construct the locus of the circle, radius ½ in., which rolls around an equilateral triangle whose altitude is 2 in. Compute to two decimal places the length of the locus and the area enclosed by it.

44. It is desired to construct a half-mile race track. The start and finish are to be straight ways intersecting at right angles at the goal. The rest of the track is to be an arc of a circle tangent to the two straight ways. Find the radius of the arc and the length of the arc in feet; also the area enclosed by the track.

45. A plot of ground consisting of two parallel straight sides and two semi-circular ends can be inscribed in a rectangle so that the straight sides of the plot lie in the longer sides of the rectangle. If the distance around the plot is ½ mi., and the longer sides of the rectangle are each 1,000 ft., find the distance between the parallel sides of the plot, and the length of those sides.

NOTES

Plane Geometry is principally concerned in:

1. Equality of lines, angles, and areas.
2. Congruent triangles, parallelograms, and circles.
3. Parallel lines, derived from equal angles.
4. Perpendicular lines, derived from equal angles.
5. Inequality of lines and angles.
6. Measurement of lines, angles, and surfaces.
7. Proportion of lines, angles, and areas.
8. Similar triangles and polygons.

All of these involve one thing — equality.

Seven theorems have varying proofs that may not fit all text-books. These seven are 35, 36, 37, 38, 61, 122, and 182. They are either proved in these notes or have a reference letter showing where a logical proof can be found.

(The letters after a note mean that a logical proof can be found as follows: W. Wentworth and Smith. Wl. Wells. S. Schultze and Sevenoak.)

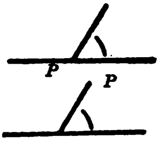
[CHAPTER I]

The theorems in this chapter may be treated informally.

7. a. See Summaries. This general axiom is one of the three basic axioms of geometry. The other two are Art. 20 and Art. 25.

20. One of the three basic axioms of geometry.

21. Two cases; when the point (P) is on the line and when the point (P) is outside the line. These are true in plane geometry from the nature of an angle.



22. This is sometimes worded, "Two straight lines cannot enclose a space." If this were true, there would be two straight lines between two points, which is impossible, by 20.

23. From the definition of a straight line, they will coincide if superposed.

24. From 23 and 7*f* and 21. Two cases.

25. This is Euclid's famous parallel postulate. It is one of the three basic axioms of geometry.

26. From the definition of supplement.

27. From the definition of supplement and of a straight angle.

[CHAPTER II]

The basic theorem for equal angles is 23, all straight angles are equal. Informal proofs of two or three statements can be used.

30. This is derived from 23 and 7*f*.

31. From 30 and 7*d*.

32. From 23 and 7*d*.

33. Similar to 32.

35. The lines cannot meet by 21. A formal proof could be used here, but this can wait till Chapter VI is begun.

36. If the angles were equal, the lines would be parallel by 35. This must be the same as the given figure by 25. A formal proof could be used here but this can wait until Chapter VII is begun.

37. From 36 and 33.

38. From 33 and 35.

[CHAPTER III]

All theorems of angle-sums depend either directly or indirectly upon Art. 26 as a basis. It is sometimes more convenient, however, to use 42.

41. Method is 26. That is, use 26.

42. Method is 26; extend one side, or draw a line through a vertex parallel to the opposite side.

This theorem is useful in proving many other theorems.

43. Method is 26; same as 42.

44. Method is 42 and 7*d*.

45. Method is 42; draw a diagonal.

47. (n–2) triangles.

48. Method is 42 and 7*c*.

49. Method is 26, 48, and 7*d*.

[CHAPTER IV]

Congruent figures will coincide if superposed, hence superposition should be attempted at first. This is necessary in the first two theorems, Arts. 52 and 53. These may be called the basic theorems of congruent triangles. In the remaining theorems of this chapter, superposition will not be used, but the conditions of each theorem will be shown to be the same as the conditions of one of the basic theorems, 52 or 53.

52. Method is superposition.

53. Method is superposition.

54. The corresponding parts of congruent triangles are equal.

55. Method is 53 and 44.

58. Method is Art. 52. The included angle is obtained by 30.

59. Method is Art. 53.

60. Method is by Art. 52 and 54; draw the bisector of the vertex angle. W; Wl; S.

Art. 60 belongs in the next chapter, but it is inserted here because it is needed in the next two theorems.

61. Adjacent position and Art. 53. Wl; S.

62. Adjacent position and Art. 52.

66. Method is 62. This is sometimes worded: To construct a triangle, having given the three sides.

67. *See* Summaries. These four theorems cover all cases of congruent triangles. Therefore, in the future, if two triangles are to be proved congruent, the conditions of the figure will be shown to be the same as the conditions of one of the above four theorems. The conclusion will then be the same.

The principal use of congruent triangles is in proving lines or angles equal. For example, if an angle x is to be proved equal to an angle y, select two triangles having x and y as corresponding parts, prove triangles congruent by *a*, *b*, *c*, or *d*, then apply 54.

If suitable triangles cannot be found, they should be formed by drawing the required lines.

[CHAPTER V]

Nearly all the proofs consist of two parts: proving triangles congruent by one of the summary, Art. 67; then the corresponding parts of congruent triangles are equal.

73. Method is Art. 60 and 7*b*.

74. Method is Art. 63 and 54; draw a perpendicular from the vertex (or the bisector of the vertex angle).

75. Method is 74 and 7*b*.

76. Method is: *1*, 63*d*; *2*, 54.

77. Method is 63*d* and 54.

78. Method is 63 and 54. This is sometimes worded: "Any point in the perpendicular at the middle of a line is equally distant from the ends of the line." S.

79. Method is 66 and 54.

80. Method is 66 and 54.

81 and *82.* Measure off the lines and angles as directed.

[CHAPTER VI]

Parallel lines lie in the same plane and do not meet no matter how far extended.

As every figure in plane geometry lies in a plane, the first part of the above definition can be neglected. It then remains to be proved that the lines cannot meet.

The first two theorems Articles 35 and 38 are fundamental. All the other theorems of this chapter depend upon one of these two.

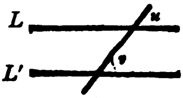


Fig. 1.

35. **Given:** $\angle u = \angle v$.
To Prove: $L \parallel L'$.
Method: Indirect.
Proof: L not $\parallel L'$, or $L \parallel L'$.

Suppose L not $\parallel L'$; then, if extended, they will meet at some point x as in fig. 2; and $\angle u \neq \angle v$. Art. 21.

This is impossible, since $\angle u = \angle v$. Given.

Therefore, $L \parallel L'$, as it is the only possibility left.

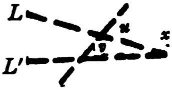


Fig. 2.

89. If a theorem is true, the converse of the opposite is always true and can be proved by the Indirect Method. Now Art. 35 is the converse of the opposite of Art. 21; for 21 may be worded, "If two lines intersect, they cannot both make equal angles with a third line." Therefore, Art. 35 is proved by the Indirect Method.

38. Method is Art. 35.

90. Method is Art. 38 or 35 (or indirect).

91. Method is Art. 38 or 35 (or indirect). W.

92. Method is Art. 35 or 38.

93. This is Euclid's famous parallel postulate. It is one of the three basic axioms of geometry.

Art. 93 makes the opposite of most parallel line theorems true. Thus: If two alternate interior angles are not equal, the lines are not parallel. Art. 37 is the converse of the opposite of this and consequently is proved by the Indirect Method. In other words, a converse theorem of parallel lines can usually be proved indirectly.

94. Method is indirect because it is the converse of the opposite of Art. 93.

101. By the def. the opposite sides must be proved parallel. This can be done by Art. 38. Draw a diagonal and prove triangles congruent by Art. 62.

102. Method is 38 (or 101). Similar to 101.

103. Method is 38 (101 or 102). Similar to 101.

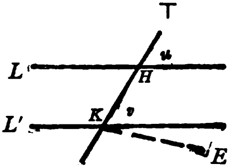
104. Method is 90 and 45.

[CHAPTER VII]

Art. 37 and 36 are fundamental for equal angles. All other cases of equal angles depend upon one of these two. Equal lines are derived from congruent triangles, Art. 67.

36. Indirect Method.

Proof: $\angle u \neq \angle v$, or $\angle u = \angle v$.
Suppose $\angle u \neq \angle u$.
Then draw KE making $\angle v = \angle HKE$.
$KE \parallel L$. Art. 35.
This is impossible since $L \parallel L'$. Art. 25.
$\angle u = \angle v$. ?



37. Method is 36 (or indirect). Wl; S.

107. Method is 37 or 36; extend two non-parallel sides until they meet.

108. Method is 45 and 32. Extend the perpendicular sides until they meet.

109. Method is: *1.* Art. 67; *2.* Art. 54; *3.* Art. 54.

110. Method is 54.

111. Method is 54.

112. Method is superposition, since it is the first case of congruent parallelograms.

113. Method is 54; draw lines, from the points on one transversal, parallel to the other transversal.

This is an important theorem, especially for proving parts of a line equal.

114. Method is 113.

116. Method is Art. 102 and Art. 109. Use the figure of Art. 106, ex. 8.

117. Method is Art. 116. Use figure of Art. 106, ex. 10. A triangle and a parallelogram are special cases of a trapezoid, and whatever is true of a trapezoid is, in general, true of a triangle or parallelogram.

The triangle case, however, is usually proved first and then the trapezoid case depends upon it.

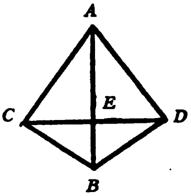
[CHAPTER VIII]

All theorems that prove lines perpendicular depend either directly or indirectly upon the definition of perpendicular, Art. 10. It is sometimes more convenient, however, to use Art. 122.

122. Method is Art. 10. Prove $\angle AEC = \angle AED$ by Art. 54.

Outline of proof:

$\triangle ACB \cong \triangle ADB$.
$\angle CAB = \angle DAB$.
$\triangle ACE \cong \triangle ADE$.
$\angle AEC = \angle AED$ and $CE = DE$.
$AB \perp CD$ and bisects CD.



123. Method is 122.
124. Method is 122.
125. Method is 122.
126. Method is Art. 10 (or indirect). It is a converse of Art. 91.
127. Method is Art. 10 or Art. 122.

[CHAPTER IX]

The basic principle of all inequality is the axiom, "The whole is greater than any one of its parts," Art. 130. All theorems of unequal angles, except 142, use this axiom as a method. All theorems of unequal lines can be developed from Art. 130 through 136, or they can be put in a class by themselves with the basis, "A straight line is the shortest line between two points," Art. 131.

134. Method is Art. 130. A most useful theorem for proving angles unequal.

135. Method is 130 and 134. On the larger line lay off a line equal to the smaller line.

136. Method is 131. On the larger angle lay off an angle equal to the smaller angle. A useful theorem.

Art. 136 is the converse of the opposite of: "If one side of a triangle is not greater than a second side, the angle opposite the first side is not greater than the angle opposite the second side." This includes two theorems, Art. 60 and 135. As both of these are true, Art. 136 can be proved by the Indirect Method.

137. Method is 131.

138. Method is 136 (or 137).

139. This is the converse of the opposite of: "If one of two lines drawn from a point in a perpendicular cuts off a not greater distance from the foot of the perpendicular than the other, then the first line is not greater than the second line." This includes two theorems, Art. 78 and 138. As both of these are true, Art. 139 can be proved by the Indirect Method.

140. Method is 136 (or 131). *140b.* Method is indirect.

141. Method is 131 (or 136).

142. This is the converse of the opposite of (52 and 141), and can be proved by the Indirect Method.

143. *See* Summaries. In proving angles unequal, *a* and *b* are the commonest methods.

144. *See* Summaries. In proving lines unequal, *b* and *c* are the commonest methods.

[CHAPTER X]

All proofs of locus theorems are in two parts depending on the definition of locus. The five fundamental locus figures are found and the last two are proved. All exercises depend upon these five figures.

152. By analysis of the definition of locus, as in 154*c* (*See* Summaries), the two statements to be proved are:

1. Any point, equally distant from the ends of a line, is on the perpendicular-bisector of that line.
2. Any point, on the perpendicular-bisector of a line, is equally distant from the ends of the line.

The first has been proved in Art. 122; the second in Art. 78.

153. The two statements to be proved are:

1. Any point, within an angle and equally distant from its sides, is on the bisector of that angle.
2. Any point, on the bisector of an angle, is equally distant from its sides.

In the first, prove a bisector by showing the angles are equal by 54. Wl.

In the second, prove the perpendiculars equal by 54.

The complete locus figure has two intersecting lines.

(In the future, a proof, unless otherwise stated, can usually be found in any standard text-book.)

[CHAPTER XI]

160. *a*, *b*, *c*, *d*, *e*, and *f* may be treated informally.

167. Find a point equally distant from the three given points by Art. 152.

168. There can be no center point because the two perpendiculars would not meet, being parallel.

169. Method is 167.

170. Method is the def. of tangent.

171. Method is Art. 140*b* (or indirect).

172. Method is Art. 54.

173. Find a point equally distant from the three sides by Art. 153.

174. Method is to draw a circle through three vertices by Art. 167, and prove that this circle passes through the fourth, etc., by Art. 54, *2*. Similarly.

It can be proved that a circle can be circumscribed about a quadrilateral if the sum of two opposite angles equals the sum of the other two angles. This figure with the triangle and regular polygon make the three cases of the circumscribed circles.

It can be proved that a circle can be inscribed in a quadrilateral if the sum of two opposite sides equals the sum of the other two sides. This figure with the triangle and regular polygon make the three cases of the inscribed circle.

[CHAPTER XII]

As equality of arcs is a new subject, the first theorem, Art. 177, is proved by superposition. The other theorems of equal arcs depend upon Art. 177. Equal chords, being straight lines, are proved by congruent triangles, Art. 67.

177. Method is superposition.

178. Method is superposition.

179. Method is Art. 177. Draw radii.

180. Method is Art. 54. Draw radii.

181. Methods are Art. 54 and Art. 177.

182. When a theorem can be divided into three cases, there is usually one case that should be proved first, and then the other cases can depend upon this. The first case has "one of each." That is, the first case here has a chord and a tangent. The method is 181. Draw a radius to the point of tangency and prove it perpendicular to the tangent and chord.

If case 2 has two tangents, draw a parallel secant and use case 1 and addition.

Then in case 3, draw a parallel tangent and use case 1 and subtraction. Wl.

183. *a* and *b.* Method is Art. 54.

184. Method is Art. 123 and 152.

185. Method is Art. 181 and 123.

[CHAPTER XIII]

Inequality of arcs, being a new subject, depends upon the axiom of inequality, Art. 130.

Inequality of chords is proved by the methods for straight lines, Art. 144.

188. Method is Art. 130. On the larger angle measure the smaller.

189. Method is Art. 130. On the larger arc measure the smaller.

190. Method is Art. 188. Draw radii.

191. Method is Art. 141. The converse of Art. 190.

192. Method is Art. 136.

193. Method is Art. 136. This is the converse of Art. 192. The outline of proof is exactly the reverse of the outline in Art. 192.

As Art. 193 is the converse of the opposite of 182*a* and 192, it can be proved indirectly.

[CHAPTER XIV]

"A central angle is measured by its intercepted arc," Art. 197, is put first as the basis. Its meaning is, the measure of a central angle equals the measure of its intercepted arc. All proofs in this chapter depend either directly or indirectly upon Art. 197, but only Art. 204 uses it as a method. The others use 204.

197. A central angle and its intercepted arc represent the same amount of revolution of a radius about the center. Therefore, measuring them by a unit of revolution, their measures would be equal. Informal treatment.

204. Three cases. One of each, — a chord and a diameter is the first case. Method is 197. Cases 2 and 3 depend upon case 1. See note on Art. 182.

205. Method is Art. 204.

206. Method is Art. 204.

207. Method is Art. 204.

208. Method is Art. 204. Draw diameter to point of tangency. (Or a line from the end of the chord parallel to the tangent.)

209. Method is 204. Draw a line joining the ends of the two chords. (Or a line from the end of one chord parallel to the other chord.)

210. Three cases. The first case is "one of each." Method is 204. Draw a line joining two intersecting points on the circumference. (Or from an intersection point parallel to the other line.) The other two cases depend upon case 1, or can be proved similarly.

211. Articles 204, 208, 209, and 210 can be all grouped under 209, if the words "within the circle" are omitted.

Starting with the vertex of the angle at the center, we have 197. Moving the vertex towards the circumference, we have 209. As the vertex approaches the circumference, one arc becomes smaller and smaller. When the vertex is on the circumference, this arc becomes zero and we have 204 and then 208. Continuing the movement of the vertex outside the circle, this arc becomes less than zero, or negative, and we then have Art. 210.

212. Method is 206.

213. Method is 206 and 170.

214. Method is 206 and 170.

215. Method is 204.

[CHAPTER XV]

The definition of regular polygon is the basis. The close relation between a regular polygon and a circle is shown here.

220. Method is: *1.* Art. 117; *2.* Art. 117.

221. Method is Art. 220, 2.

225. Method is Arts. 220 and 177.

226. Method is Arts. 220 and 177.

227. Method is Arts. 185 and 220.

230. Inscribe a regular hexagon and show that its perimeter equals 6; circumscribe a square and show that its perimeter equals 8. Then use Art. 229*a*.

234. The methods of constructing one and only one triangle are Art. 233, exercises 1, (2 and 4), 3, and 5. This includes the construction of right triangles from the conditions of Art. 63.

235. *See* Summaries. It is customary to name the lines of a triangle by small letters having certain relations to the capital letters at the vertices.

Thus: a is the side opposite $\angle A$; b is opposite $\angle B$; c is opposite $\angle C$; h_a is the altitude to side a; t_a is the bisector of $\angle A$; m_c is the median to c; etc.

Also, *R* is the radius of the circumscribed circle; *r* the radius of the inscribed circle.

236. Ex. 9. There may be no figure; one figure; two figures.

[CHAPTER XVI]

"Three or more parallel lines intercept proportional parts on two transversals," Art. 237, is put first as the basis of all proportion of lines. Its more common form, however, is that of Art. 245. All proportion in this chapter depends upon one of these two.

237. Commensurable case only. Method is Art. 113.

245. Method is 237 (or similar to 237).

247. Method is 245.

248. Method is 237.

249. Method is 245. Make a line parallel to the base of a triangle.

250. The smaller part in external division is algebraically negative. Thus, if the dividing point *C* in internal division of line *AB* is moved towards *A*, *CA* becomes smaller; if *C* reaches *A*, *CA* becomes zero; if *C* goes beyond *A*, *CA* becomes less than zero or negative. *CB* continually grows larger, but the sum of the two parts *CA* and *CB* always equals *AB*.

251. Method is 245. Make a line parallel to the base of a triangle.

252. Method is indirect. The converse of Art. 245. (*See* 93.)

254. *See* Summaries. If it is impossible to prove the equality of the two desired ratios at first, prove each ratio equal to a third ratio and then use substitution (Art. 7*b*).

[CHAPTER XVII]

The basic theorem is Art. 257 although Art. 258 can be treated as basic. All the other theorems of this chapter depend upon one of these two. The arrangement of the chapter is almost identical with that of Chapter IV on congruent triangles, the first two theorems using a form of superposition, and the last two adjacent position.

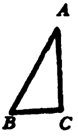
257. Method is Art. 253; a form of superposition.

258. Method is Art. 257 or Art. 253; a form of superposition. This theorem is basic if Art. 253 is used for the method.

259. The corresponding sides of similar triangles are in proportion and the corresponding angles are equal.

261. Method is Art. 257.

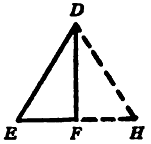
262. Method is Art. 258.



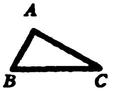
263. **Given:** $\frac{AB}{DE} = \frac{AC}{DF}$; rt. $\triangle ABC$ and DEF.

To prove: $\triangle ABC \sim \triangle DEF$.

Method: Adjacent position and Art. 257.
At D construct $\angle FDH = \angle A$; extend EF to H.



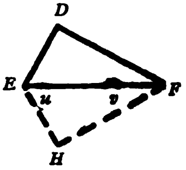
Proof:
$\triangle ABC \sim DFH$ Art. 257.
$\frac{AB}{DH} = \frac{AC}{DF}$ Art. 259.
$\frac{AB}{DE} = \frac{AC}{DF}$ Given.
$DH = DE$?
$\angle E = \angle H$ Art. 60.
$\triangle DFH \cong \triangle DEF$ Art. 63.
$\triangle ABC \sim \triangle DEF$.



264. **Given:** $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$.

To prove: $\triangle ABC \sim \triangle DEF$.

Method: Adjacent position and Art. 257.
At E construct $\angle u = \angle B$; at F construct $\angle v = \angle C$.



Proof:
$\triangle ABC \sim \triangle HEF$ Art. 257.
$\frac{AB}{EH} = \frac{BC}{EF}$ Art. 259.
$\frac{AB}{DE} = \frac{BC}{EF}$ Given.
$EH = DE$?
$FH = DF$?
$\triangle HEF \cong \triangle DEF$. Art. 62.
$\triangle ABC \sim \triangle DEF$.

265. This theorem is the basis of trigonometry.

266. Method is Art. 257.

267. The principal use of similar triangles is to prove four lines in proportion.

A pair of corresponding sides are always opposite a pair of corresponding equal angles.

[CHAPTER XVIII]

The proof of lines in proportion from similar triangles has two parts: *1.* Proving triangles similar by one of the summary, Art. 267; *2.* The corresponding sides of similar triangles are in proportion, Art. 259. The proof of the product of two lines equals the product of two other lines, has three parts: *1.* Proving triangles similar; *2.* Corresponding sides in proportion; *3.* Product of the means equals product of the extremes, Art. 242.

269. Method is 259. The same thing can be proved of medians and angle-bisectors.

271. Method is 242. Join the ends of two chords.

272. Method is 242. Join the alternate intersections.

273. Method is 242. Join the intersection points.

274. The last three theorems can be grouped under one statement: If two chords intersect (internally or externally), the product of the parts of one chord equals the product of the parts of the other chord.

276. Method is: *1.* Art. 265; *2.* Art. 259; *3.* Art. 259.

277. Method is 276, 2. Make a right triangle.

278. Method is 277 (or 276, 3).

[CHAPTER XIX]

This chapter is given up entirely to the Pythagorean theorem and its applications. The method of proof for the theorem is Art. 276, 3. Draw a perpendicular from the right angle to the hypotenuse.

[CHAPTER XX]

Similar polygons follow the definition, Art. 253. They depend usually upon similar triangles. Circles follow regular polygons.

285. Method is Art. 253. Use similar triangles.

286. Method is 267*b*. Use similar polygons, Art. 253.

287. Method is Art. 285 (or 253).

288. Method is Art. 253.

289. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, then $\frac{a+c+e}{b+d+f} = \frac{a}{b}$. *See* Proportion, 8.

290. Method is Arts. 253 and 289.

291. Method is Arts. 290 and 269. Draw radii and apothem.

293. The theorem of limits is to be treated informally.

There are two parts to the condition: *1.* The variables are *always* equal. *2.* Each approaches a limit. Under these conditions, their limits are equal.

294. Measurement of a circle is proved from the measurement of a regular polygon by means of 293. Method is 291 and 293. Inscribe regular polygons with the same number of sides.

295. Method is Art. 294: $\frac{C}{D} = \frac{C^1}{D^1}$.

296. $\frac{C}{D} = \pi$, whence $C = 2 \pi R$. An important formula.

297. Method is Art. 230. The value of π cannot be found exactly. Its commonly accepted value is 3.1416−, or nearly $3\frac{1}{7}$. Compared to our number system, π is incommensurable.

[CHAPTER XXI]

"The area of a rectangle equals the product of the base and altitude," Art. 300, is put first as the basis. All other theorems of area depend either directly or indirectly upon this. The circle follows the regular polygon.

300. This may be taken as a definition, or its commensurable case may be treated informally by dividing it into unit squares.

301. Method is 300.

302. Method is 300. Erect perpendiculars at the ends of the base. Show that the triangles formed are congruent; then the parallelogram equals the rectangle.

303. Method is 302. Make a parallelogram from the triangle.

304. Method is 303. Draw a diagonal making triangles.

As the mid-line of a trapezoid equals one-half the sum of the bases (Art. 117), Art. 304 can be read, "The area of a trapezoid equals the product of the altitude and the mid-line."

305. The area of any polygon can be found by dividing into triangles, and measuring the triangles.

306. Method is 303. Draw all the radii.

307. Method is 306 and 293. Circumscribe a regular polygon about the circle.

308. As area of a circle $= \frac{1}{2} C \times R$, and $C = 2 \pi R$,

Area of a circle $= \pi R^2$. An important formula.

309. A sector is to its circle its central angle is to four right angles. The area of a segment of a circle is found by subtracting the area of the triangle formed by the chord and the radii from the area of the sector. This can be done if the radius is known and the central angle is 30°, 45°, 60°, 90°, and their supplements; also 36° and 72°.

[CHAPTER XXII]

Equality of areas can be developed from congruent triangles. Another method is to show that their area formulas are equal. This is easily done in the first four theorems and Art. 322. The other theorems can depend upon the first four.

312. Method is 302. Another proof is by Art. 52. *See* 302.

313. Method is 303.

314. Method is 304. *See* note. (Or by Art. 52.)

315. Method is 302 and 303.

316. This is the Pythagorean theorem again, but proved here by areas. The previous proof was algebraic. The method is Arts. 52 and 315.

317. Method is 316.

318. Method is 316.

319. Method is 317.

320. Method is 313.

321. The algebraic expression a^2 means in geometry the area of a square whose side is a. The algebraic expression ab means in geometry the area of a parallelogram whose base is b and altitude a.

322. Method is 277 and 321.

[CHAPTER XXIII]

The theorems of this chapter depend upon the corresponding theorems of area, except 331, 332, 333, and 334. These four depend either directly or indirectly upon 328.

325. Method is 302.

326. Method is 302 (or 325).

327. Method is 302 (or 325).

328. Method is 303.

329. Method is 303 (or 328).

330. Method is 303 (or 328).

331. Method is 328. Draw corresponding altitudes.

332. Method is 328. Draw corresponding altitudes. Another method is Art. 331.

333. Method is Art. 332 and 289. Draw corresponding diagonals.

334. Method is Art. 333 and 269.

335. Method is Art. 308 (or 334 and 293).

[CHAPTER XXIV]

The Pythagorean theorem, Art. 282, is the basis.

339. Method is Art. 282.

340. Method is Art. 282.

341. Arts. 282, 339, and 340 can be grouped under one statement: The square of one side of a triangle equals the sum of the squares of the other two sides plus twice the product of one of those sides and the external projection of the other upon it.

Thus if we draw an obtuse triangle, we have Art. 340. If the vertex of the triangle be moved to the right or left, the external projection becomes smaller. When this external projection becomes zero, we have Art. 282. Continuing this movement the external projection becomes less than zero or negative. We then have Art. 339.

342. Method is 339 and 340.

343. Method is 270.

344. Method is 270. Circumscribe a circle about the triangle.

345. Method is 303. Draw the altitude.

346. Method is 282.

347. Method is 346.

[CHAPTER XXV]

Extreme and Mean Ratio; Square Ratio; Incommensurable Cases.

351. Method is Art. 273.

352. Method is Art. 351. Angles of 36°, 60°, and 90°, their halves, their sums and differences, are practically the only angles that can be constructed by straight edge and compass.

353. Method is Art. 352.

354. Method is 326 and 322.

355. Method is 354 and 287.

356. Method is 320 and 247.

358. Method is 177 and 293.

359. Method is 237 and 293.

360. Method is 300 and 293.

SUMMARIES OF METHODS

ARRANGED ALPHABETICALLY IN TWO GROUPS, GENERAL AND GEOMETRICAL

GENERAL

235. **Analysis of Construction Problems:**

Construction problems usually depend upon the drawing of locus lines or of a triangle. One and only one triangle can be constructed under the conditions of congruent triangles. When the construction is not clear, it is often advisable to:

1. Draw the figure as if the construction was completed.

2. Mark all known lines and angles.

3. From these known parts, construct some part of the figure. (Usually a triangle.)

4. Around this figure, complete the construction.

Analysis of Numerical Exercises:

1. Draw the figure.

2. Mark all known parts with their numbers.

3. Recall the theorem that connects the known parts with the desired part by an equation.

4. Write and solve this equation.

156. **Analysis of Theorems:**

1. Read the statement carefully, separating the given conditions from the thing to be proved.

If the statement is in the form of a conditional sentence, the condition is given and the conclusion is to be proved.

Thus, *If two sides of a triangle are equal,* | *the opposite angles are equal.*

If the statement is in the form of a declarative sentence, the subject and its modifiers form the given conditions; the rest of the statement is to be proved.

Thus, *Two straight lines perpendicular to a third straight line* | *are parallel.*

2. Having determined the thing to be proved, recall the methods of proving that thing.

3. Select the method most suited to the given conditions.

4. Build up the proof according to the method, beginning with the given conditions and ending with the desired conclusion.

7. **General Axioms of Equality:**

a. The whole equals the sum of all of its parts.
b. Equals may be substituted for equals in an equation or inequality.
c. If equals are added to equals, the results are equal.
d. If equals are subtracted from equals, the results are equal.
e. If equals are multiplied into equals, the results are equal.
f. If equals are divided into equals, the results are equal.

General Axioms of Inequality:

130. The whole is greater than any one of its parts.
132. *a.* If equals are added to, or subtracted from, unequals, the results are unequal in the same order.
b. If positive equals are multiplied, or divided, into unequals the results are unequal in the same order.
c. If the first of three quantities is greater than the second and the second greater than the third, then the first is greater than the third.
d. If unequals are added to unequals in the same order, the results are unequal in the same order.
e. If unequals are subtracted from equals, the results are unequal in reverse order.

89. **Indirect Method of Proof:**

In the indirect method, the desired thing is not proved, but everything else is shown to be impossible. Then the desired thing must be true.

In detail the steps are:

1. State all the possibilities in regard to the thing to be proved.

2. Suppose one of the undesired possibilities to be true, and show that these conditions lead to something impossible. This case is then thrown out.

3. Repeat with each undesired possibility.

4. Finally, there is nothing left but the desired possibility, which must be true without further proof.

16. **Superposition:**

Superposition is the first test for congruency, and has three main steps:

Arcs unequal:

194. *a.* The greater of two central angles intercepts the greater arc.
b. The greater of two chords subtends the greater arc.

Areas equal:

323. *a.* Congruent triangles or parallelograms.
b. Prove their area-formulas equal.
c. Parallelograms or triangles are equal, if they have equal bases and equal altitudes.

Areas in proportion:

336. *a.* Divide their area-formulas.
b. Parallelograms or triangles are to each other as the products of their bases and altitudes.
c. Two similar polygons are to each other as the squares of any two corresponding lines. (332–335.)

Formulas:

In lettering the parts of a triangle, a is the side opposite angle A, etc.; h_a is the altitude to a; t_a is the bisector of angle A; m_a is the median to a; r_i is the radius of the inscribed circle; r_c is the radius of the circumscribed circle; S is the area; s is the semi-perimeter; b is the base; b' is the upper base.

50. Sum of the angles of a polygon. $S = (n-2)$ st. ∠s.

One angle of a regular polygon. $A = \frac{n-2}{n}$ st. ∠s.

Areas:

310.	Parallelogram	$s = bh.$
	Triangle	$s = \frac{1}{2} bh.$
	Trapezoid	$s = \frac{1}{2}h\,(b + b')$
	Regular polygon .	$s = \frac{1}{2} pr.$
	Circle	$s = \frac{1}{2} cr = \pi r^2;\ c = 2\pi r.$

Right Triangle:

283. Hypotenuse $c^2 = a^2 + b^2.$

If $A = 60°$, $c = 2b$; $a = \frac{c}{2}\sqrt{3}.$

If $A = 45°$, $a = b$; $c = a\sqrt{2}.$

Equilateral Triangle:

$$h = \frac{a}{2}\sqrt{3}.$$

$$s = \tfrac{1}{2}ah = \frac{a^2}{4}\sqrt{3}.$$

$$r_i = \tfrac{1}{3}h = \frac{a}{6}\sqrt{3}.$$

$$r_c = \tfrac{2}{3}h = \frac{a}{3}\sqrt{3}.$$

Oblique Triangle:

348. $$h_a = \frac{2}{a}\sqrt{s(s-a)(s-b)(s-c)}$$

$$S = \tfrac{1}{2}ah = \sqrt{s(s-a)(s-b)(s-c)}$$

$$r_i = \frac{S}{s}$$

$$r_c = \frac{bc}{2h_a}$$

$$m_a = \tfrac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$$

$$t_a = \frac{2}{b+c}\sqrt{bcs(s-a)}$$

Line passes through a given point:

154. *e*, *1.* The line is the locus of points having the conditions of the given point.

2. Indirect.

Loci:

154. *a.* **To find the locus** of points under given conditions:

Make many points under the given conditions, until the line through these points is evident.

b. **To find a point** under two given conditions:

Find the locus line for the first condition, then the locus line for the second condition; and the intersection of these locus lines determines the point under both conditions.

c. **To prove that a line is the locus** of points under certain conditions, prove:

- *1.* Any point fulfilling the conditions is on the line.
- *2.* Any point on the line fulfills the conditions.

Or, one of these, and the opposite of the other.

Polygons, regular:

231. *a.* All sides are equal and all angles equal.
b. It is formed by chords or tangents at the points of equal division of a circle.
c. To inscribe a regular polygon in a circle, place its central angle at the center of the circle.

Polygons, similar:

298. *a.* Their corresponding angles are equal and their corresponding sides are in proportion.
b. They are composed of the same number of triangles, similar each to each and similarly placed.

Straight lines determined:

28. *a.* Only one straight line can be drawn through two given points.
b. Through a given point, only one straight line can be drawn, making a given angle with a given straight line, and having this angle on a given side of each line.
c. Through a given point, only one straight line can be drawn perpendicular to a given straight line.
d. Through a given point, only one straight line can be drawn parallel to a given straight line.

Straight lines equal:

85. *a.* The corresponding parts of congruent triangles are equal.
b. If two angles of a triangle are equal, the opposite sides are equal.
118. *c.* Opposite sides of a parallelogram.
d. Diagonals of a parallelogram bisect each other.
e. Diagonals of a rectangle are equal.
f. Parallel lines cutting off equal parts on one transversal cut off equal parts on any other transversal.
186. *g.* Equal arcs are subtended by equal chords.
h. Chords equally distant from the center are equal.
i. Two tangents from an outside point are equal.
j. A radius perpendicular to a chord bisects the chord.

Straight lines parallel:

105. *a.* They lie in the same plane and do not meet no matter how far extended.
b. Alternate-interior angles equal.
c. Corresponding angles equal.
d. Each perpendicular to the same line.
e. Each parallel to the same line.
f. Two interior angles on the same side of the transversal supplementary.
g. Opposite sides of a parallelogram.
h. The line joining the middle points of the non-parallel sides of a trapezoid or of a triangle is parallel to the base.

105. A quadrilateral is a parallelogram if:
1. The opposite sides parallel.
2. The opposite sides equal.
3. Two sides equal and parallel.
4. The diagonals bisect each other.

255. *i.* A line dividing two sides of a triangle proportionally is parallel to the third side.

Straight lines perpendicular:

128. *a.* They form equal adjacent angles.
b. They form a right angle.
c. A line perpendicular to one of two parallel lines is perpendicular to the other.
d. Two points, each equidistant from the ends of a straight line, determine the perpendicular bisector of that line.
e. The diagonals of a rhombus (or a square) bisect each other at right angles.
145. *f.* The shortest line from a point to a straight line is the perpendicular.
175. *g.* A tangent to a circle is perpendicular to the radius drawn to the point of tangency.
218. *h.* An angle inscribed in a semicircle is a right angle.

Straight lines in proportion:

254. *a.* Parallel lines cut off proportional parts on two transversals.
b. The bisector of an angle of a triangle divides the opposite side into parts proportional to the other two sides.

279. *c.* The corresponding sides of similar triangles are in proportion.
d. The corresponding parts of two chords intersecting internally or externally are in inverse proportion.

Straight lines in one straight line:

28. *a.* If the sum of two adjacent angles equals two right angles, their exterior sides lie in one straight line.

Straight lines tangent to a circle:

175. *a.* The line touches the circle at only one point.
b. The line is perpendicular to a radius at its end.

Straight lines unequal:

144. *a.* The whole is greater than any one of its parts.
b. A straight line is the shortest line between two points.
c. If two angles of a triangle are unequal, the opposite sides are unequal and the greater side is opposite the greater angle.
d. If two lines are drawn from a point in a perpendicular, cutting off unequal distances from the foot of the perpendicular, the more remote is the greater.
e. The perpendicular is the shortest line from a point to a line.
f. The sum of two lines, drawn from a point to the ends of a line, is greater than the sum of two other lines similarly drawn but enveloped by them.
g. If two triangles have two sides of the one equal respectively to two sides of the other but the included angle of the first greater than the included angle of the second, the third side of the first is greater than the third side of the second.
195. *h.* Unequal arcs are subtended by unequal chords.
i. The chord nearer the center is the greater.

Triangles congruent:

67. *a.* If two triangles have two sides and the included angle of the one equal respectively to two sides and the included angle of the other, the triangles are congruent.
b. If two triangles have two angles and a side of the one equal respectively to two angles and the corresponding side of the other, the triangles are congruent.

c. If two triangles have three sides of the one equal respectively to three sides of the other, the triangles are congruent.

d. If two right triangles have a side and other part, except the right angle, of the one equal respectively to a corresponding side and other part, except the right angle, of the other, the triangles are congruent.

Triangles similar:

267. *a.* Two angles of the one equal respectively to two angles of the other.

b. Two pairs of sides in proportion and the included angles equal.

c. Three pairs of sides in proportion.

d. In right triangles, an acute angle of the one equals an acute angle of the other, or if two sides of the one are in proportion with two corresponding sides of the other.

DEFINITIONS

A

Acute angle. An angle less than a right angle.

Adjacent angles. Two angles that have a common vertex and a common side between them.

Alternate-interior angles. When two lines in the same plane are crossed by a transversal, two interior angles on opposite sides of the transversal, but not adjacent.

Alternation. *See* Proportion.

Altitude. A perpendicular from a vertex to the opposite side.

Angle. Two lines that meet form an angle. An angle represents the amount of revolution of one side about the vertex.

Antecedent. *See* Proportion.

Apothem. The perpendicular from the center to the side of a regular polygon.

Arc. A part of a circumference.

Area. The ratio of a surface to a unit of surface.

Arm. A side adjacent to the right angle in a right triangle.

Axiom. A statement assumed to be true without proof.

B

Base. That side of a figure upon which it apparently rests.

Bisect. To divide into two equal parts.

C

Central angle. In a circle, an angle between two radii; in a regular polygon, an angle between two successive radii.

Chord. A straight line joining two points in a circumference.

Circle. A figure fromed by a curved line, called a **circumference,** all points of which are equally distant from a point within called the center.

Circumference. *See* Circle.

Circumscribe. A circle is circumscribed about a polygon when it passes through every vertex; the polygon is said to be inscribed in the circle.

A polygon is circumscribed about a circle when its sides are tangents to the circle; the circle is said to be inscribed in the polygon.

Coincide. All points in one figure fall upon corresponding points in a second figure.

Commensurable. Exactly divisible by a common unit of measure.

Complement. An angle is the complement of a second angle when their sum equals a right angle.

Composition. *See* Proportion.

Congruent. Two figures that have their corresponding parts equal. They have the same form and size, and can be made to coincide.

Consequent. *See* Proportion.

Constant. A quantity whose position, form, or size does not change during the discussion.

Converse theorem is formed when the condition and conclusion of a theorem change places.

When the condition and conclusion are made negative and change places, the **converse of the opposite is formed.**

Corresponding. Similarly placed.

Corollary. A dependent theorem.

D

Degree is $\frac{1}{360}$ of a circumference.

Determine. A line is determined in position when it is the only line that can be drawn under the given conditions.

Diameter. A chord through the center of a circle.

Distance between two points is represented by a straight line; between a point and a straight line, by a perpendicular; between a point and a circle is the external part of the straight line from the point to the center of the circle.

Division. A line can be divided into two parts internally or externally:

Internally when the point of division is in the line.

Externally when the point of division is in the line extended.

In either case the parts made by the division point are the distances from this point to the ends of the given line. In external division the parts are algebraic not geometric.

Division. *See* Proportion.

E

Equal figures have the same size.

In the case of straight lines, angles, and circles, they will coincide if superposed.

Equiangular triangle has all angles equal.

Equilateral triangle has all sides equal.

Exterior angle of a triangle is formed by the extension of one side.

Extreme and mean ratio. A line is divided in extreme and mean ratio when one part of the line is a mean proportional between the whole line and the other part.

Extremes. *See* Proportion.

F

Figure. *See* Geometrical figure.

G

Geometrical figure. Any combination of points, lines, surfaces, and solids.

Geometry is the science of space. It treats of the position, form, and size of geometrical figures.

H

Homologous. *See* Corresponding.

Hypotenuse. The side opposite the right angle in a right triangle.

Hypothesis. The given conditions.

I

Incommensurable. Not exactly divisible by a common unit of measure.

Inscribe. *See* Circumscribe.

Inversion. *See* Proportion.

Isosceles triangle has two sides equal.

L

Limit. *See* Variable.

Line. The boundary of a surface. It has length but no width or thickness.

Locus of points under given conditions is the line or lines that contain all points under these conditions and no others.

M

Mean proportional. *See* Proportion.

Means. *See* Proportion.

Measure. *See* Measurement.

Measurement of a quantity is the process of finding the numerical value or measure of that quantity.

A unit of measure can be anything but it must be of the same kind as the quantity to be measured. Thus, inch, etc., are units of measure for straight lines; square inch, etc., for surfaces; degree, etc., for revolutions.

The number of times a unit of measure is contained in a given quantity is the **numerical measure** of that quantity.

Median. A straight line from a vertex of a triangle to the middle of the opposite side.

Mid-line. A straight line joining the mid-points of the two non-parallel sides of a trapezoid.

O

Obtuse angle. An angle greater than a right angle but less than a straight angle.

Opposite theorem is formed when the condition and conclusion of a theorem are both made negative.

P

Parallel lines. Two straight lines, in the same plane, that never meet no matter how far extended.

Parallelogram. A quadrilateral with opposite sides parallel.

Perimeter. The sum of the sides of a closed figure.

Perpendicular. A straight line, meeting another straight line, making two adjacent angles equal. Each angle is called a **right angle.**

π. The ratio between a circumference of a circle and its diameter. Its value is approximately $3\frac{1}{7}$.

Plane surface is such that a straight line joining any two points therein lies wholly in the surface.

Point is the boundary or end of a line. It has position but no length, width, or thickness.

Polygon. A portion of surface bounded by lines.

Postulate. A geometrical axiom of construction.

Problem. A statement of a figure to be constructed.

Product of means. *See* Proportion.

Projection of a point upon a line is the foot of the perpendicular from the point to the line. The projection of a line upon a line is the line through the feet of all the perpendiculars from the first line to the second line; that is, the locus of the projections of its points.

Proportion. An equality between two ratios. Thus $\frac{a}{b} = \frac{c}{d}$.

A proportion may be changed in form by algebraic operations:

1. Alteration $\frac{a}{c} = \frac{b}{d}$.

2. Inversion $\frac{b}{a} = \frac{d}{c}$.

3. Composition $\frac{a+b}{a \text{ or } b} = \frac{c+d}{c \text{ or } d}$.

4. Division $\frac{a-b}{a \text{ or } b} = \frac{c-d}{c \text{ or } d}$.

5. Composition and division $\frac{a+b}{a-b} = \frac{c+d}{c-d}$.

6. Product of the means equals the product of the extremes, $bc = ad$.

7. If the product of two quantities equals the product of two other quantities, either pair may be made the means of a proportion of which the other pair are the extremes. (Converse of 6.)

8. In a series of equal ratios, the sum of the antecedents is to the sum of the consequents as any antecedent is to its consequent.

The first and fourth terms of a proportion are called the **extremes**; the second and the third, the **means**. When the second and third terms are equal, either is called the **mean proportional**, and the last term is the **third proportional**.

The first and third terms are called **antecedents**; the second and fourth, **consequents**.

Proposition. A theorem or problem.

Q

Quadrilateral. A portion of surface bounded by four lines.

R

Radius. In a circle, a straight line from the center to the circumference. In a regular polygon, the straight line from the center to any vertex.

Ratio of two quantities is found by dividing their numerical measures.

Rectangle. A parallelogram with all angles right angles.

Rectilinear. Made with straight lines.

Regular polygon. A polygon with all sides equal and all angles equal.

Revolution. The moving of a straight line in a plane, about its end as an axis, until it reaches its original position.

Rhombus. An equilateral parallelogram.

Right angle. Half a straight angle. *See* Perpendicular.

Right triangle. A triangle having one angle a right angle.

S

Scalene triangle. A triangle having no sides equal.

Secant. A straight line cutting a circle in two points.

Sector. Part of a circle between two radii and their intercepted arc.

Segment. Part of a circle between a chord and its arc, or between two parallel chords.

Series of equal ratios. *See* Proportion.

Similar polygons have corresponding angles equal and corresponding sides in proportion. (They have the same form.)

Solid. An enclosed portion of space. It has length, width, and thickness.

Square. An equilateral rectangle.

Straight angle. An angle with its sides in a straight line and on opposite sides of the vertex.

Straight line. Any part will coincide with any other equal part, if superposed.

Subtend. To stretch beneath.

Supplement. One angle is the supplement of a second angle, if their sum equals a straight angle.

Surface is the boundary of a solid. It has no thickness, only length and width.

Synthetic proof. A series of statements beginning with the given condition and ending with the desired conclusion. One statement leads to another, and every statement depends upon a previous definition, axiom, or theorem.

T

Tangent. A line that lies in the plane of a circle and touches it at one and only one point no matter how far extended.

Theorem. A statement to be proved.

Third proportional. *See* Proportion.

Transversal. A line crossing one or more lines.

Trapezoid. A quadrilateral having two sides parallel.

Triangle. A portion of surface bounded by three lines.

U

Unit of measure. *See* Measurement.

V

Variable quantity changes its size or form or both. The special variable usually considered in Geometry is a continually increasing or continually decreasing variable. This variable approaches but never reaches a constant quantity called the **limit.**

Vertex. The point of meeting of two sides of an angle. In a triangle, the point opposite the base.

Vertical angles. The opposite angles made by two intersecting lines.

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